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ORIGINAL SCIENTIFIC PAPER OPEN ACCESS

An analytical approach for free vibration analysis of Euler-Bernoulli stepped beams with axial-bending coupling effect

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ABSTRACT

Free vibration of eccentrically stepped beams with one step change in cross-section is considered. It is assumed that the longitudinal symmetry axes of the beam segments are translationally shifted along the vertical direction with respect to each other. The effect of that arrangement of the segments on the coupling of axial and bending vibrations of the stepped beam is analyzed. The beam segments are modeled in the frame of the Euler-Bernoulli theory of elastic beams. Two numerical examples are presented.

KEYWORDS

Stepped beam, Step eccentricity, Axial-bending coupling, Euler-Bernoulli theory, Natural frequency, Free vibration

1. INTRODUCTION

Stepped beams often appear as an integral part of various devices and structures in mechanical and civil engineering. That is why the buckling and vibration analysis of stepped beams is extremely important for engineering practice. Despite the large number of papers published in connection with this problem (see e.g. [1-22]), this field of scientific research is still actual with intensive development and generation of new scientific problems. In the available literature, there are mainly references dealing with homogeneous stepped beams. Also, tapered beams can be modeled as stepped beams in a manner described in [23, 24]. In the recent years stepped beams made of functionally graded materials represent an actual research field due to special mechanical characteristics of this kind of materials [25-27]. Decoupled axial and bending vibrations are mainly considered for stepped beams. However, often these two types of vibrations can be coupled. The reason for this may be, for example, the shape of cross-section of beams [28, 29], rigid bodies eccentrically attached to the stepped beams [23, 30-33], angled-beam joints in the frame structures [31, 34] and the compliant mechanisms [35], varying material characteristics of beam segments through their thickness direction [36] as well as mutual eccentric positions of longitudinal symmetry axes of stepped beams segments [22].

The last-mentioned cause of coupling is considered in this paper. In this sense, the objective of our paper is to extend the approach described in [31, 33] to the case of Euler-Bernoulli eccentrically stepped beams with one step change in cross-section. In the frame of the Euler-Bernoulli theory of elastic beams, to the authors' best knowledge of the literature, the appearance of axial-bending coupling effect in the case of this type of stepped beams was not considered.

2. FORMULATION OF GOVERNING EQUATIONS

An eccentrically stepped beam of rectangular cross-section with one step discontinuity is shown in Figure 1. The width and thicknesses of cross-sections of segments (S_1) and (S_2) are denoted by $b_i\,(i=1,2)$ and $h_i\,(i=1,2)$, respectively. In the undeformed configuration of the stepped beam, local stationary inertial coordinate frames $\{x_i,y_i,z_i\}(i=1,2)$ are placed in the manner shown in Figure 1. Also, the longitudinal symmetry axes of segments (S_1) and (S_2) are translationally shifted in the vertical direction by an amount e. The quantities $u_i(z_i,t)(i=1,2)$ and $w_i(z_i,t)(i=1,2)$ represent the axial and transverse displacements, respectively, of any point of the neutral axes of the beam segments. The material and geometric characteristics of segments (S_1) and (S_2) are: E_i is the modulus of elasticity, $I_{x(i)}$ is the cross-sectional area moment of inertia about axis x_i, A_i is the cross-sectional area, ρ_i is the mass density, and L_i is the length of the i-th beam segment. The partial differential equations of bending and axial free vibrations of segments (S_1) and (S_2) are as follows [37, 38]:

$$E_i I_{x(i)}\frac{\partial^4 w_i(z_i,t)}{\partial z_i^4}+\rho_i A_i\frac{\partial^2 w_i(z_i,t)}{\partial t^2}=0,\ \ i=1,2, \tag{1}$$

$$E_i A_i\frac{\partial^2 u_i(z_i,t)}{\partial z_i^2}-\rho_i A_i\frac{\partial^2 u_i(z_i,t)}{\partial t^2}=0,\ \ i=1,2. \tag{2}$$

Based on the method of separation of variables [37, 38], the displacements $u_i(z_i,t)(i=1,2)$ and $w_i(z_i,t)(i=1,2)$ can be written as:

$$w_i(z_i,t)=W_i(z_i)T(t),\qquad u_i(z_i,t)=U_i(z_i)T(t), \tag{3}$$

where $U_i(z_i)(i=1,2)$ and $W_i(z_i)(i=1,2)$ are the mode shapes in free axial and bending vibrations, respectively, and $T(t)=e^{i\omega t}$, $i=\sqrt{-1}$, and ω is the natural angular frequency of vibration of the stepped beam.

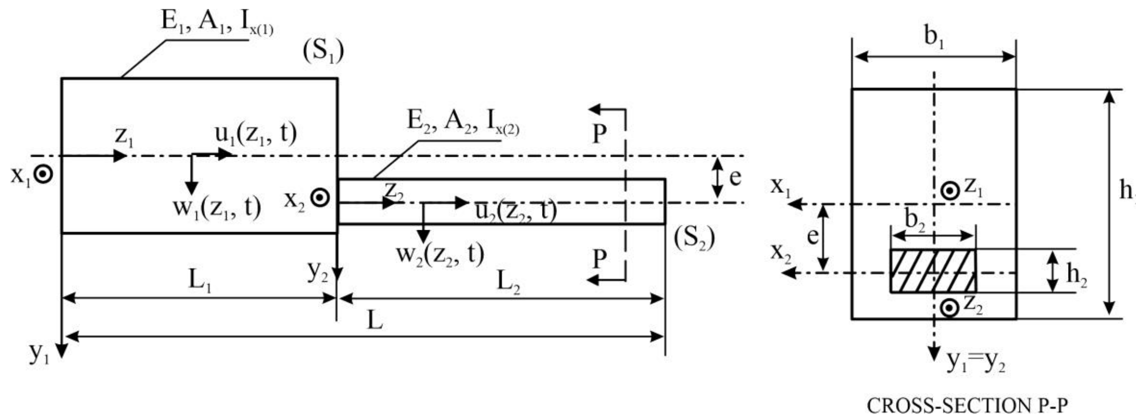

Figure 1: Stepped Euler-Bernoulli beam

Introducing (3) into (1) and (2) yields:

$$\frac{d^4 W_i(z_i)}{dz_i^4}-k_i^4 W_i(z_i)=0,\ \ i=1,2, \tag{4}$$

$$\frac{d^2 U_i(z_i)}{dz_i^2}+p_i^2 U_i(z_i)=0,\ \ i=1,2, \tag{5}$$

$$\frac{d^2\mathrm{T}(\mathrm{t})}{dt^2}+\omega^2\mathrm{T}(\mathrm{t})=0, \tag{6}$$

where:

$$k_i^4=\frac{\rho_i A_i}{E_i I_{x(i)}}\omega^2,\qquad p_i^2=\frac{\rho_i}{E_i}\omega^2,\ \ i=1,2. \tag{7}$$

Based on (7), the following relation can be established:

$$p_i = \sqrt{\frac{I_{x(i)}}{A_i}} k_i^2, \quad i = 1,2. \tag{8}$$

Combining (7) and (8) with the following expressions:

$$k_1 = k, \quad p_1 = \sqrt{\frac{I_{x(1)}}{A_1}} k^2 , \tag{9}$$

yields:

$$k_2 = \sqrt[4]{\frac{E_1 I_{x(1)} \rho_2 A_2}{E_2 I_{x(2)} \rho_1 A_1}} k, \quad p_2 = \sqrt{\frac{E_1 I_{x(1)} \rho_2}{E_2 A_1 \rho_1}} k^2 , \tag{10}$$

$$\omega^2 = \frac{E_1 I_{x(1)}}{\rho_1 A_1} k^4. \tag{11}$$

Introducing now the following dimensionless quantities:

$$\bar{z}_i = \frac{z_i}{L}, \ \bar{W}_i(\bar{z}_i) = \frac{W(L\bar{z}_i)}{L}, \quad \bar{U}_i(\bar{z}_i) = \frac{U(L\bar{z}_i)}{L}, \quad \frac{d^j}{d^j} = \frac{1}{L^j}\frac{d^j}{d\bar{z}^j}, \ j = 1,2,\ldots, \tag{12}$$

the equations (4) and (5) as well as the relations (9) and (10) can be written in the following dimensionless forms:

$$\frac{d^4 \bar{W}_i(\bar{z}_i)}{d\bar{z}_i^4} - \bar{k}_i^4 \bar{W}_i(\bar{z}_i) = 0, \ i = 1,2, \tag{13}$$

$$\frac{d^2 \bar{U}_i(\bar{z}_i)}{d\bar{z}_i^2} + \bar{p}_i^2 \bar{U}_i(\bar{z}_i) = 0, \ i = 1,2, \tag{14}$$

$$\bar{k}_1 = \bar{k}, \ \ \bar{p}_1 = \bar{r}\bar{k}^2, \tag{15}$$

$$\bar{k}_2 = \sqrt[4]{\frac{\gamma_A \gamma_\rho}{\gamma_E \gamma_I}} \bar{k}, \quad \bar{p}_2 = \sqrt{\frac{\gamma_\rho}{\gamma_E}} \bar{r}\bar{k}^2, \tag{16}$$

where $\bar{k}_i = k_i L$, $\bar{p}_i = p_i L$, $\gamma_\rho = \rho_2 / \rho_1$, $\gamma_E = E_2 / E_1$, $\gamma_A = A_2 / A_1$, $\gamma_I = I_{x(2)} / I_{x(1)}$, $\bar{r} = \sqrt{I_{x(1)} / (A_1 L^2)}$, and:

$$\bar{k} = kL, \qquad \bar{\omega} = \bar{k}^2 = \sqrt{\frac{\rho_1 A_1 L^4}{E_1 I_{x(1)}}} \omega \tag{17}$$

are the dimensionless frequency coefficient and the dimensionless natural angular frequency, respectively. General solutions of the equations (13) and (14) are given as follows [37, 38]:

$$\bar{W}_i(\bar{z}_i) = C_{1(i)} \cos(\bar{k}_i \bar{z}_i) + C_{2(i)} \sin(\bar{k}_i \bar{z}_i) + C_{3(i)} \cosh(\bar{k}_i \bar{z}_i) + C_{4(i)} \sinh(\bar{k}_i \bar{z}_i), \quad i = 1,2, \tag{18}$$

$$\bar{U}_i(\bar{z}_i) = C_{5(i)} \cos(\bar{p}_i \bar{z}_i) + C_{6(i)} \sin(\bar{p}_i \bar{z}_i), \quad i = 1,2, \tag{19}$$

where $C_{1(i)},\ldots,C_{6(i)}$ are integration constants.

3. BOUNDARY CONDITIONS AND THE FREQUENCY EQUATION

3.1. Boundary conditions at the left end of the stepped beam

For the clamped left end of the considered stepped beam the following boundary conditions hold:

$$\bar{U}_1(0) = 0, \quad \bar{W}_1(0) = 0, \quad \frac{d\bar{W}_1}{d\bar{z}_1}(0) = 0, \tag{20}$$

whereas for the pinned left end one has:

$$\bar{U}_1(0) = 0, \quad \bar{W}_1(0) = 0, \quad \frac{d^2\bar{W}_1}{d\bar{z}_1^2}(0) = 0. \tag{21}$$

[1] Introducing $\mathbf{C}_1 = \left[C_{1(1)} \ldots C_{6(1)}\right]^T$ as a vector of integration constants corresponding to segment (S_1) and putting (18) and (19) into (20) and (21) yields the following matrix relation:

$$\mathbf{C}_1 = \mathbf{T}_0 \mathbf{C}_0 \tag{22}$$

where for the clamped end one has:

$$\mathbf{C}_0 = \left[C_{1(1)} \;\; C_{2(1)} \;\; C_{6(1)}\right]^T, \quad \mathbf{T}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{23}$$

and for the pinned one:

$$\mathbf{C}_0 = \left[C_{2(1)} \;\; C_{4(1)} \;\; C_{6(1)}\right]^T, \quad \mathbf{T}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{24}$$

3.2. Boundary conditions at the junction of the segments

In order to establish corresponding continuity conditions at the junction of the stepped beam segments, let us consider an infinitesimal part of the stepped beam at the step location as it is depicted in Figure 2.

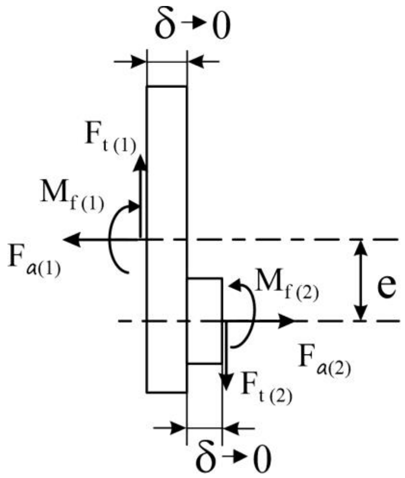

Figure 2: Free-body diagram of an infinitesimal part of the stepped beam at the junction of segments

Here, $F_{t(1)}$ and $F_{t(2)}$ are the shear forces defined as [31, 33]:

$$F_{t(1)} = -E_1 I_{x(1)} \frac{d^3 W_1}{dz_1^3}(L_1 - \delta), \qquad F_{t(2)} = -E_2 I_{x(2)} \frac{d^3 W_2}{dz_2^3}(0 + \delta), \tag{25}$$

$F_{a(1)}$ and $F_{a(2)}$ are the axial forces given as [31, 33]:

$$F_{a(1)} = E_1 A_1 \frac{dU_1}{dz_1}(L_1 - \delta), \qquad F_{a(2)} = E_2 A_2 \frac{dU_2}{dz_2}(0 + \delta), \tag{26}$$

and, finally, $M_{f(1)}$ and $M_{f(2)}$ are the bending moments defined as [31, 33]:

$$M_{f(1)} = -E_1 I_{x(1)} \frac{d^2 W_1}{dz_1^2}(L_1 - \delta), \qquad M_{f(2)} = -E_2 I_{x(2)} \frac{d^2 W_2}{dz_2^2}(0 + \delta). \tag{27}$$

Taking the length δ approaches to zero yields the following continuity conditions at the level of forces and moments:

$$F_{t(1)} = F_{t(2)} \Leftrightarrow \frac{d^3\bar{W}_1}{d\bar{z}_1^3}(\gamma_{L1}) = \gamma_E\gamma_I \frac{d^3\bar{W}_2}{d\bar{z}_2^3}(0), \tag{28}$$

$$F_{a(1)} = F_{a(2)} \Leftrightarrow \frac{\mathrm{d}\bar{U}_1}{d\bar{z}_1}(\gamma_{L1}) = \gamma_E\gamma_A \frac{\mathrm{d}\bar{U}_2}{d\bar{z}_2}(0), \tag{29}$$

$$M_{f(1)} = M_{f(2)} + F_{a(2)}\mathrm{e} \Leftrightarrow \frac{d^2\bar{W}_1}{d\bar{z}_1^2}(\gamma_{L1}) = \gamma_E\gamma_I \frac{d^2\bar{W}_2}{d\bar{z}_2^2}(0) - \frac{\gamma_E\gamma_A\bar{e}}{\bar{r}^2}\frac{\mathrm{d}\bar{U}_2}{d\bar{z}_2}(0), \tag{30}$$

where $\gamma_{L1} = L_1 / L$ and $\bar{e} = e / L$. Also, at the level of displacements one has the following continuity conditions:

$$W_1(L_1) = W_2(0) \Leftrightarrow \bar{W}_1(\gamma_{L1}) = \bar{W}_2(0), \tag{31}$$

$$\frac{dW_1}{dz_1}(L_1) = \frac{dW_2}{dz_2}(0) \Leftrightarrow \frac{d\bar{W}_1}{d\bar{z}_1}(\gamma_{L1}) = \frac{d\bar{W}_2}{d\bar{z}_2}(0), \tag{32}$$

$$U_1(L_1) - \mathrm{e}\frac{dW_1}{dz_1}(L_1) = U_2(0) \Leftrightarrow \bar{U}_1(\gamma_{L1}) - \bar{e}\frac{d\bar{W}_1}{d\bar{z}_1}(\gamma_{L1}) = \bar{U}_2(0). \tag{33}$$

The continuity conditions (28)-(33) generate the following matrix relation:

$$\mathbf{T}_{1L}\mathbf{C}_1 = \mathbf{T}_{1R}\mathbf{C}_2 \tag{34}$$

where $\mathbf{C}_2 = \left[C_{1(2)} \cdots C_{6(2)}\right]^T$ is the vector of integration constants corresponding to segment (S_2) and entries of the matrix $\mathbf{T}_{1L} \in R^{6\times6}$ are:

$$\boldsymbol{T}_{1L} = \begin{bmatrix} sin(\bar{k}\gamma_{L1}) & -cos(\bar{k}\gamma_{L1}) & sinh(\bar{k}\gamma_{L1}) & cosh(\bar{k}\gamma_{L1}) & 0 & 0 \\ 0 & 0 & 0 & 0 & -sin(\bar{r}\bar{k}^2\gamma_{L1}) & cos(\bar{r}\bar{k}^2\gamma_{L1}) \\ -cos(\bar{k}\gamma_{L1}) & -sin(\bar{k}\gamma_{L1}) & cosh(\bar{k}\gamma_{L1}) & sinh(\bar{k}\gamma_{L1}) & 0 & 0 \\ cos(\bar{k}\gamma_{L1}) & sin(\bar{k}\gamma_{L1}) & cosh(\bar{k}\gamma_{L1}) & sinh(\bar{k}\gamma_{L1}) & 0 & 0 \\ -sin(\bar{k}\gamma_{L1}) & cos(\bar{k}\gamma_{L1}) & sinh(\bar{k}\gamma_{L1}) & cosh(\bar{k}\gamma_{L1}) & 0 & 0 \\ \bar{e}\bar{k}\,sin(\bar{k}\gamma_{L1}) & -\bar{e}\bar{k}\,cos(\bar{k}\gamma_{L1}) & -\bar{e}\bar{k}\,sinh(\bar{k}\gamma_{L1}) & -\bar{e}\bar{k}\,cosh(\bar{k}\gamma_{L1}) & cos(\bar{r}\bar{k}^2\gamma_{L1}) & sin(\bar{r}\bar{k}^2\gamma_{L1}) \end{bmatrix}, \tag{35}$$

and of the matrix $\mathbf{T}_{1R} \in R^{6\times6}$:

$$\mathbf{T}_{1R} = \begin{bmatrix} 0 & -\gamma_E\gamma_I\sqrt[4]{\left(\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}\right)^3} & 0 & \gamma_E\gamma_I\sqrt[4]{\left(\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}\right)^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_E\gamma_A\sqrt{\frac{\gamma_\rho}{\gamma_E}} \\ -\gamma_E\gamma_I\sqrt{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}} & 0 & \gamma_E\gamma_I\sqrt{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}} & 0 & 0 & -\frac{\gamma_E\gamma_A\bar{e}}{\bar{r}}\sqrt{\frac{\gamma_\rho}{\gamma_E}} \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & \sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}} & 0 & \sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{36}$$

Solving (34) for $\mathbf{C}_2$ yields:

$$\mathbf{C}_2 = \mathbf{T}_1\mathbf{C}_1 \tag{37}$$

where the matrix $\mathbf{T}_1 \in R^{6\times6}$ is determined by:

$$\mathbf{T}_1 = \mathbf{T}_{1R}^{-1}\mathbf{T}_{1L} \tag{38}$$

and represents the transfer matrix between the integration constants of segments (S_1) and (S_2).

3.3. Boundary conditions at the right end of the stepped beam

In this section the following types of the right end of the stepped beam will be considered: free end, clamped end, and pined end. The boundary conditions for free right end read:

$$\frac{d\bar{U}_2}{d\bar{z}_2}(\gamma_{L2})=0,\quad \frac{d^2\bar{W}_2}{d\bar{z}_2^2}(\gamma_{L2})=0,\quad \frac{d^3\bar{W}_2}{d\bar{z}_2^3}(\gamma_{L2})=0, \tag{39}$$

for clamped right end one has:

$$\bar{U}_2(\gamma_{L2})=0,\quad \bar{W}_2(\gamma_{L2})=0,\quad \frac{d\bar{W}_2}{d\bar{z}_2}(\gamma_{L2})=0, \tag{40}$$

and, finally, the corresponding boundary conditions for pinned right end are:

$$\bar{U}_2(\gamma_{L2})=0,\quad \bar{W}_2(\gamma_{L2})=0,\quad \frac{d^2\bar{W}_2}{d\bar{z}_2^2}(\gamma_{L2})=0, \tag{41}$$

where $\gamma_{L2}=L_2/L=1-\gamma_{L1}$. Introducing (18) and (19) into (39)-(41) yields the following matrix expression:

$$\mathbf{T}_2\mathbf{C}_2=0_{3\times1} \tag{42}$$

where $0_{3\times1}\in R^{3\times1}$ is a zero matrix and the matrix $\mathbf{T}_2\in R^{3\times6}$ has the following entries:

- free right end

$$\mathbf{T}_2=\begin{bmatrix} 0 & 0 & 0 & 0 & -\sin\left(\sqrt{\frac{\gamma_\rho}{\gamma_E}}\bar{r}\bar{k}^2\gamma_{L2}\right) & \cos\left(\sqrt{\frac{\gamma_\rho}{\gamma_E}}\bar{r}\bar{k}^2\gamma_{L2}\right) \\ -\cos\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & -\sin\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \cosh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \sinh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & 0 & 0 \\ \sin\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & -\cos\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \sinh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \cosh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & 0 & 0 \end{bmatrix} \tag{43}$$

- clamped right end

$$\mathbf{T}_2=\begin{bmatrix} 0 & 0 & 0 & 0 & \cos\left(\sqrt{\frac{\gamma_\rho}{\gamma_E}}\bar{r}\bar{k}^2\gamma_{L2}\right) & \sin\left(\sqrt{\frac{\gamma_\rho}{\gamma_E}}\bar{r}\bar{k}^2\gamma_{L2}\right) \\ \cos\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \sin\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \cosh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \sinh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & 0 & 0 \\ -\sin\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \cos\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \sinh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \cosh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & 0 & 0 \end{bmatrix} \tag{44}$$

- pinned right end

$$\mathbf{T}_2=\left[\begin{matrix} 0 & 0 & 0 \\ \cos\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \sin\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \cosh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) \\ -\cos\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & -\sin\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & \cosh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) \end{matrix}\right.$$

$$\begin{array}{ccc} 0 & \cos\left(\sqrt{\frac{\gamma_\rho}{\gamma_E}}\bar{r}\bar{k}^2\gamma_{L2}\right) & \sin\left(\sqrt{\frac{\gamma_\rho}{\gamma_E}}\bar{r}\bar{k}^2\gamma_{L2}\right) \\ \sinh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & 0 & 0 \\ \sinh\left(\sqrt[4]{\frac{\gamma_\rho\gamma_A}{\gamma_E\gamma_I}}\bar{k}\gamma_{L2}\right) & 0 & 0 \end{array}\Bigg] \tag{45}$$

3.4. Derivation of the frequency equation

Substituting (22) and (37) into (42) implies a homogeneous system of equations for unknown components of the vector $\mathbf{C}_0$. This equations system can be written in the matrix form as follows:

$$\mathbf{TC}_0 = 0_{3\times1}, \tag{46}$$

where $\mathbf{T} \in R^{3\times3}$ represents overall transfer matrix given as:

$$\mathbf{T} = \mathbf{T}_2\mathbf{T}_1\mathbf{T}_0 . \tag{47}$$

Finally, the corresponding frequency equation for the problem analyzed reads:

$$f\left(\bar{k}\right) \equiv \det\mathbf{T} = 0 . \tag{48}$$

4. NUMERICAL EXAMPLES

In numerical calculations of this section, the stepped beam geometrical parameters given in [21] will be used as follows: $h_1 = 19.05\,\text{mm}$, $h_2 = 5.49\,\text{mm}$, $b_1 = b_2 = 25.4\,\text{mm}$, $L_1 = 254\,\text{mm}$, $L_2 = 140\,\text{mm}$. Based on the theoretical considerations given in Sections 2 and 3, the effect of eccentricity e on dimensionless natural angular frequencies of the stepped beam for various combinations of materials of the beam segments is shown in Table 1.

Table 1: Values of the lowest four dimensionless natural angular frequencies for various combinations of materials of beam segments

Boundary conditions	γ_E	γ_ρ	$\bar{e}$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_4$
C-F	1	1	0	4.91792	11.5118	41.2372	63.3360
			$(h_1-h_2)/2L$	4.91721	11.5160	41.1452	63.2388
	210/70	7800/2702	0	3.56078	12.7782	38.3359	68.4161
			$(h_1-h_2)/2L$	3.55984	12.7772	38.0964	68.1344
	210/200	7800/5700	0	4.37158	10.8834	38.1153	59.7993
			$(h_1-h_2)/2L$	4.37092	10.8816	38.0313	59.6232
	70/210	2702/7800	0	6.04735	10.2739	44.1409	58.7212
			$(h_1-h_2)/2L$	6.04700	10.2732	44.1087	58.6712
	200/210	5700/7800	0	5.47852	12.1880	44.2492	67.5286
			$(h_1-h_2)/2L$	5.47778	12.1873	44.1545	67.4851
C-C	1	1	0	10.5456	41.6661	63.4959	126.956
			$(h_1-h_2)/2L$	10.9146	42.6224	63.8636	127.547
	210/70	7800/2702	0	12.1451	38.6258	68.5107	122.251
			$(h_1-h_2)/2L$	12.2614	39.4791	68.6165	122.755
	210/200	7800/5700	0	10.2694	38.5099	59.8737	117.221
			$(h_1-h_2)/2L$	10.6321	39.2554	60.4155	117.596
	70/210	2702/7800	0	9.36794	44.6403	58.9085	131.053
			$(h_1-h_2)/2L$	9.73649	45.1417	59.3185	131.361
	200/210	5700/7800	0	10.7400	44.6716	67.8257	135.238
			$(h_1-h_2)/2L$	11.1144	45.8394	68.0187	136.001
C-P	1	1	0	8.48564	33.1612	55.5437	111.304
			$(h_1-h_2)/2L$	9.03568	33.5446	56.4792	111.553
	210/70	7800/2702	0	8.52040	32.2248	57.7435	110.564
			$(h_1-h_2)/2L$	8.80366	32.7437	58.1751	110.898
	210/200	7800/5700	0	8.11873	30.2376	53.2552	101.200
			$(h_1-h_2)/2L$	8.65478	30.5024	54.2718	101.308

	70/210	2702/7800	0	2.90797	33.6863	54.0219	110.938
			$(h_1-h_2)/2L$	8.8912	33.7989	54.8195	111.008
	200/210	5700/7800	0	8.77674	36.4176	57.7648	121.942
			$(h_1-h_2)/2L$	9.33593	36.9565	58.5750	122.396
P-P	1	1	0	3.58639	27.5587	45.8283	101.296
			$(h_1-h_2)/2L$	3.99912	28.4686	46.1990	102.027
	210/70	7800/2702	0	4.97231	25.0462	50.0108	96.9531
			$(h_1-h_2)/2L$	5.08686	25.8629	50.1198	97.5953
	210/200	7800/5700	0	3.53750	25.3954	43.2920	93.5887
			$(h_1-h_2)/2L$	3.93047	26.1356	43.7984	94.0675
	70/210	2702/7800	0	2.29499	30.0517	42.0169	105.198
			$(h_1-h_2)/2L$	2.83477	30.5298	42.4227	105.562
	200/210	5700/7800	0	3.60704	29.7006	48.7503	108.041
			$(h_1-h_2)/2L$	4.03779	30.7882	48.9768	109.005

The limit value $\bar{e}=(h_1-h_2)/(2L)$ corresponds to the case of an eccentrically stepped beam with the flat bottom surface. The influence of the eccentricity $\bar{e}$ on the lowest four dimensionless frequency coefficients is examined in Figures 3, 4, 5, and 6. At that, the values of $\bar{e}$ is taken from the interval $0.001 \le \bar{e} \le (h_1-h_2)/(2L)$.

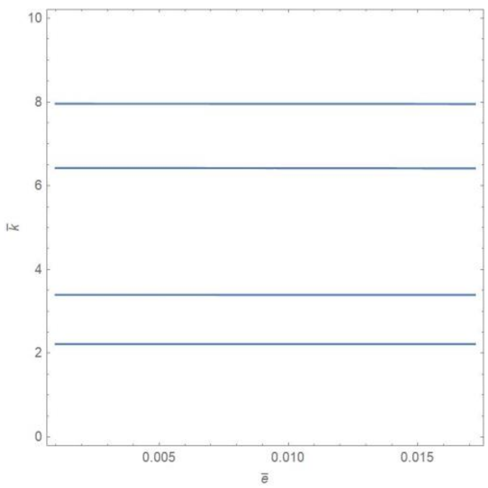

Figure 3: The effect of the eccentricity $\bar{e}$ on the lowest four dimensionless frequency coefficients of the clamped-free stepped beam

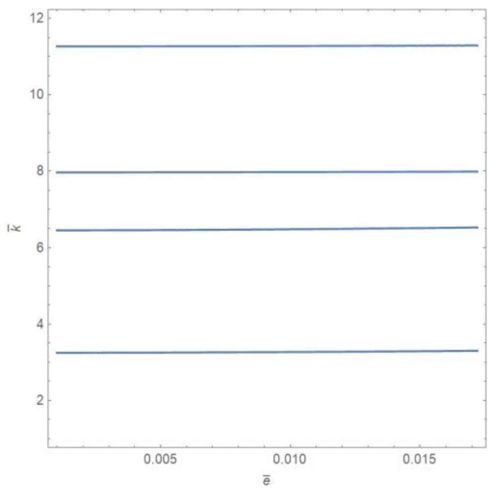

Figure 4: The effect of the eccentricity $\bar{e}$ on the lowest four dimensionless frequency coefficients of the clamped-clamped stepped beam

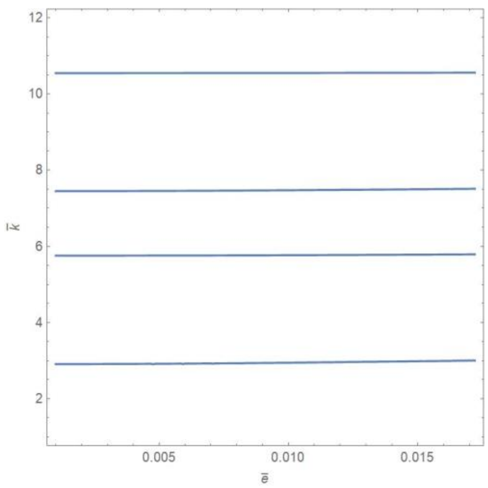

Figure 5: The effect of the eccentricity $\bar{e}$ on the lowest four dimensionless frequency coefficients of the clamped-pinned stepped beam

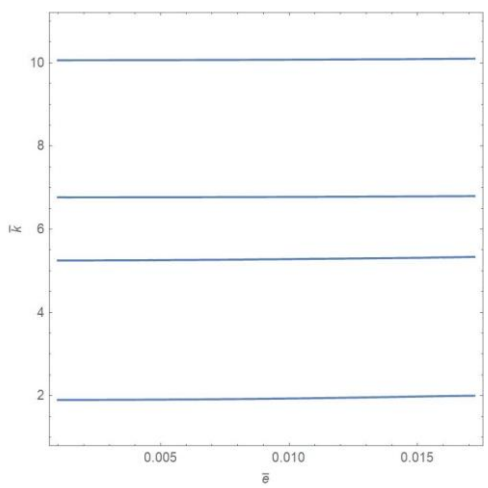

Figure 6: The effect of the eccentricity $\bar{e}$ on the lowest four dimensionless frequency coefficients of the pinned-pinned stepped beam

5. CONCLUSIONS

An analytical approach based on the transfer matrix method for free vibration analysis of stepped Euler-Bernoulli beams with coupled axial and bending vibrations has been presented. The mutual eccentric position of the beam segments longitudinal axes has been considered as the cause of coupling of axial and bending vibrations. The presented method can be also used in the cases of pure axial and pure bending vibrations of stepped beams. The numerical simulations show that the existence of eccentricity e causes small changes in the values of the first four natural frequencies. These changes are more pronounced in the case of the beam segments made of different materials. By changing the value of eccentricity e, the crossing and veering phenomena have not been detected.

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ORIGINAL SCIENTIFIC PAPER OPEN ACCESS

Bending, buckling, and free vibration analysis of multi-span beams

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ABSTRACT

In this paper, the modified segmented rod method (MSRM) will be used for bending, buckling, and free vibration analysis of multi-span beams. Since there is MSRM for a one-span beam in literature, slight corrections will enable the application of this method to the case of multi-span beams. The motivation for this research lies in the fact that there are engineering objects that must be modeled as multi-span beams, such as bridges. The proposed MSRM uses the absolute coordinates of the rigid segments, so the introduction of different boundary conditions as well as the loads can be done easily. The effectiveness of the MSRM was tested through numerical examples where the comparison with current results from the available literature was done.

KEYWORDS

Multi-span beams, Bending, Buckling, Free vibration, Modified segmented rod method

1. INTRODUCTION

Beams as the basic element of many constructions are the subject of research even today. As structural elements in planar models, they are predominantly loaded with axial forces and bending couplings, so it is important to calculate static deformations during bending, critical values of axial forces to prevent buckling, but also to consider the characteristics of the beam during free vibrations. The largest number of studies available in the literature refer to single-span beams. On the other hand, there are continuous systems with repeated beam elements that can be modeled as multi-span beams [1]. Bridges are a typical example of this type of object. The tendency is for bridges to be thinner than before, due to the saving of materials, so they are more sensitive to static and dynamic loads and their analysis in this regard is increasingly important. The literature dealing with this type of beam is plenty. References [2-6] indicate that the topic is still current. In this paper, the modified segmented rod method (MSRM in the further) will be used for the static and dynamic analysis of multi-span beams. This method is developed and described in detail in [7] for the case of a single-span beam with various boundary conditions. Due to the use of absolute coordinates of the endpoints, the results obtained in [7] can be used here with minimal corrections.

2. MSRM MODEL OF THE BEAMWITH ARBITRARY LOADS AND BOUNDARY CONDITIONS

Figure 1(*a*) shows the MSRM model of the beam with arbitrary loads and boundary conditions which consist of n elementary beam segments of length $\Delta L=L/n$. It was assumed that the beam is generally loaded at points k (k=1,…,n+1) by the transversal forces F_k and q_k, and force coupling M_k. Note that the force

$$q_k = q\Delta L \frac{p}{p+1}, \tag{1}$$

represents an approximately calculated force that comes from a continuous load, where q is the unit load and p represents the number of elementary beam segments of length ΔL on which the load is distributed.

Each of the elastic segments is further discretized into rigid segments connected by elastic rotational hinges, as shown in Figure 1(b) and explained in detail in reference [7].

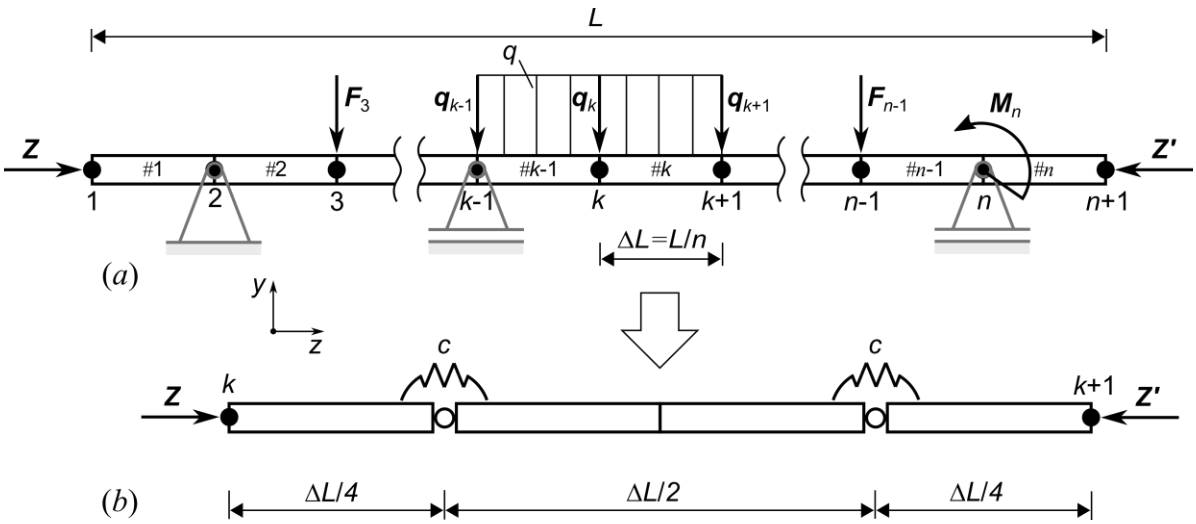

Figure 1: MSRM model [7] of the beam with arbitrary loads and boundary conditions

The differential equations of motion of the considered MSRM model of the free-free beam were given in reference [7]. Relying on them, and taking into account the arbitrary loads that act at the node points k(qk, Fk, and Mk), the differential equations of motion of the considered model have the form:

$$\mathbf{M}\ddot{\mathbf{v}} + \left(\mathbf{K} - Z\mathbf{S}\right)\mathbf{v} = \mathbf{Q}_{\mathbf{v}}, \tag{2}$$

where

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{11,1} & \mathbf{X}_{12,1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_{21,1} & \mathbf{X}_{22,1}+\mathbf{X}_{11,2} & \mathbf{X}_{12,2} & \mathbf{0} & \vdots & \vdots & \vdots \\ \mathbf{0} & \vdots & \ddots & \vdots & \mathbf{0} & \vdots & \vdots \\ \vdots & \mathbf{0} & \mathbf{X}_{21,k-1} & \mathbf{X}_{22,k-1}+\mathbf{X}_{11,k} & \mathbf{X}_{12,k} & \mathbf{0} & \vdots \\ \vdots & \vdots & \mathbf{0} & \ddots & \ddots & \vdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \mathbf{X}_{21,n-1} & \mathbf{X}_{21,n-1} & \mathbf{X}_{22,n-1}+\mathbf{X}_{11,n} & \mathbf{X}_{12,n} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_{21,n} & \mathbf{X}_{22,n} \end{bmatrix} \in R^{2(n+1)\times 2(n+1)}, \tag{3}$$

and $\mathbf{X} = \mathbf{M}, \mathbf{K}, \mathbf{S}$.

The block matrices, which make up the matrices M, K, and S, are given in the explicit form in reference [7] and read:

$$\mathbf{M}_{11,k} = \frac{\rho A \Delta L}{192} \begin{bmatrix} 80 & 14\Delta L \\ 14\Delta L & 3(\Delta L)^2 \end{bmatrix}, \tag{4}$$

$$\mathbf{M}_{12,k} = \left(\mathbf{M}_{21,k}\right)^T = \frac{\rho A \Delta L}{192} \begin{bmatrix} 16 & -4\Delta L \\ 4\Delta L & -(\Delta L)^2 \end{bmatrix}, \tag{5}$$

$$\mathbf{M}_{22,k} = \frac{\rho A \Delta L}{192} \begin{bmatrix} 80 & -14\Delta L \\ -14\Delta L & 3(\Delta L)^2 \end{bmatrix}, \tag{6}$$

$$\mathbf{K}_{11,k} = \frac{c}{2(\Delta L)^2} \begin{bmatrix} 16 & 8\Delta L \\ 8\Delta L & 5(\Delta L)^2 \end{bmatrix}, \tag{7}$$

$$\mathbf{K}_{12,k} = \left(\mathbf{K}_{21,k}\right)^T = \frac{c}{2(\Delta L)^2}\begin{bmatrix} -16 & 8\Delta L \\ 8\Delta L & 5(\Delta L)^2 \end{bmatrix}, \tag{8}$$

$$\mathbf{K}_{22,k} = \frac{c}{2(\Delta L)^2}\begin{bmatrix} 16 & -8\Delta L \\ -8\Delta L & 5(\Delta L)^2 \end{bmatrix}, \tag{9}$$

$$\mathbf{S}_{11,k} = \frac{c}{8\Delta L}\begin{bmatrix} 16 & 4\Delta L \\ 4\Delta L & 3(\Delta L)^2 \end{bmatrix}, \tag{10}$$

$$\mathbf{S}_{12,k} = \left(\mathbf{S}_{21,k}\right)^T = \frac{c}{8\Delta L}\begin{bmatrix} -16 & 4\Delta L \\ -4\Delta L & (\Delta L)^2 \end{bmatrix}, \tag{11}$$

$$\mathbf{S}_{22,k} = \frac{c}{8\Delta L}\begin{bmatrix} 16 & -4\Delta L \\ -4\Delta L & 3(\Delta L)^2 \end{bmatrix}. \tag{12}$$

Vector of coordinates of points $k(k=1,\ldots,n+1)$reads:

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1^T & \mathbf{v}_2^T & \ldots & \mathbf{v}_k^T & \ldots & \mathbf{v}_n^T & \mathbf{v}_{n+1}^T \end{bmatrix}^T \in \mathrm{R}^{2(n+1)\times 1}, \tag{13}$$

whereas $\mathbf{v}_k = \begin{bmatrix} y_k & \varphi_k \end{bmatrix}^T$ consists of transversal displacement y_k of point k and rotation angle φ_k at this point.

The vector of generalized coordinates corresponding to the previously defined vector **v** is of the form:

$$\mathbf{Q}_{\mathbf{v}} = \begin{bmatrix} \mathbf{Q}_{\mathbf{v}_1}{}^T & \mathbf{Q}_{\mathbf{v}_2}{}^T & \ldots & \mathbf{Q}_{\mathbf{v}_k}{}^T & \ldots & \mathbf{Q}_{\mathbf{v}_n}{}^T & \mathbf{Q}_{\mathbf{v}_{n+1}}{}^T \end{bmatrix}^T \in \mathrm{R}^{2(n+1)\times 1}, \tag{14}$$

where

$$\mathbf{Q}_{\mathbf{v}_k} = \begin{bmatrix} \xi_{F_k} \cdot F_k + \xi_{q_k} \cdot q_k \,, \, \xi_{M_k} \cdot M_k \end{bmatrix}^T, \tag{15}$$

and it holds that each of the coefficients ξ_{F_k}, ξ_{q_k}, or ξ_{M_k} has a value of 1 if a corresponding load is acting at the point k, otherwise it is equal to 0. Since the differential equations of the observed model have been formed for the case of a free-free beam, the introduction of boundary conditions should now be considered. As stated in reference [7], the introduction of different boundary conditions in MSRM is simple. It is only necessary to remove the rows and columns in the matrices **M**, **K**, and **S**, as well as the rows of vector $\mathbf{Q}_\mathbf{v}$, corresponding to the constrained displacements. So, if the beam is pinned at point k then $y_k=0$, or if it is clamped then $y_k=0$ and $\varphi_k=0$.

2.1. Eigenvalue analysis

If we consider free vibrations of the axially loaded beam, then the eigenvalue problem that corresponds to differential equations (2) reads:

$$\left(\mathbf{K} - Z\mathbf{S} - \omega^2\mathbf{M}\right)\mathbf{u} = \mathbf{0}_{2(n+1)\times 1}. \tag{16}$$

By solving the characteristic determinant

$$\left|\mathbf{K} - Z\mathbf{S} - \omega^2\mathbf{M}\right| = \mathbf{0}_{2(n+1)\times 1}, \tag{17}$$

the natural frequencies ω_r ($r=1,\ldots, 2(n+1)$) of the considered beam will be determined.

The eigenvector associated with the eigenvalue ω_r, reads:

$$\mathbf{u}_r = \begin{bmatrix} \mathbf{u}_{1,r}^T & \mathbf{u}_{2,r}^T & \ldots & \mathbf{u}_{k,r}^T & \ldots & \mathbf{u}_{n,r}^T & \mathbf{u}_{n+1,r}^T \end{bmatrix}^T \in \mathrm{R}^{2(n+1)\times 1}, \tag{18}$$

where

$$\mathbf{u}_{k,r} = \begin{bmatrix} y_{k,r} & \varphi_{k,r} \end{bmatrix}^T. \tag{19}$$

Now, the coordinates of the mode shape points read $\{z_k; y_{k,r}\}$ ($k=1,\ldots, n+1$), where $z_k=(k-1)\Delta L$. By connecting all these points, while respecting the boundary conditions, the approximate r-th mode shape of the beam will be obtained.

2.2. Buckling analysis

Assume now that the beam is loaded only by the axial force Z. When the value of the axial force Z reaches a critical value, the beam will buckle. At that moment, the value of the natural frequency ω of free beam vibration is equal to zero. If we consider that ω=0, the characteristic equation becomes:

$$\left|\mathbf{K}-Z\mathbf{S}\right| = \mathbf{0}_{2(n+1)\times 1}. \tag{20}$$

By solving this characteristic equation, one can obtain 2(n+1) values for Z. The smallest one represents the critical buckling force Z_{cr}. By following the procedure shown for obtaining the mode shapes of free beam vibrations, it is also possible to obtain here the buckling shapes of the beam.

2.3. Bending analysis

Let us now assume that only static loads Fk, qk, or Mk(k=1,…,n+1) act on the beam and that the axial force Z is equal to zero. Also, the assumption is that there are no vibrations under the effect of static loads, so ω=0. If we apply the above assumptions, the differential equation (2) becomes algebraic and takes the form:

$$\mathbf{K}\cdot\mathbf{v} = \mathbf{Q}_{\mathbf{v}}. \tag{21}$$

Now, the vector of deformation of the beam reads:

$$\mathbf{v} = \mathbf{K}^{-1}\mathbf{Q}_{\mathbf{v}}. \tag{22}$$

Finally, the coordinates of the deflection line of the beam read {z_k; y_k}, (k=1,…,n+1) where z_k=(k-1)ΔL. By connecting all these points, while respecting the boundary conditions, the approximate shape of the deflection line will be obtained.

3. NUMERICAL EXAMPLES

In the following examples, a beam with a square cross-section of dimensions 0.02×0.02 m will be used, with the following material characteristics: Young's modulus E=2.1×10^{11}Pa and the material density ρ=7800 kg/m^3.The length of the beam L will be given in each of the examples separately. The results of the finite element method (FEM in the further) will be provided by using the Ansys software. The number of finite elements n will be calculated as the quotient of the total length L of the beam and the smallest dimension of the cross-section (it is 0.02m here) because the beam is modeled as a line body.

3.1. Example 1

Figure 2 shows the four-span pinned-pinned beam. The length of all beam spans is equal whereas the total beam length reads L=2 m. The first five dimensionless natural frequencies $(\omega^2\rho AL^4/(EI_x))^{1/4}$ are computed by the proposed MSRM and by FEM, with n=100 elastic segments used, and compared with the exact results in the closed form obtained in reference [2]. All of these values, including the relative errors between the results of MSRM and FEM to the exact ones placed in parentheses, are shown in Table 1.

The values of the natural frequencies obtained by MSRM are in excellent agreement with the exact values. The largest relative error of 0.023% for the case of the fourth natural frequency proves it. Moreover the first frequency obtained by the MSRM achieves an exact value. The results obtained by the FEM by using the same number of elastic segments n are slightly worse than the results of the MSRM method. So, the largest relative error 0.529% is at the fifth frequency and the smallest 0.125% at the first frequency. Although MSRM has a slight advantage, the results of both methods converge to the exact values quickly enough.

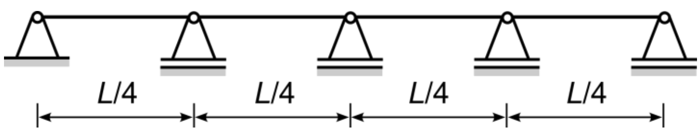

Figure 2: P-P beam with four equally spaced spans

Table 1: The first five dimensionless natural frequencies of P-P beam with four equally spaced spans

Mode number	*n*=100		Exact [2]
	MSRM	FEM	(K=10^{12})
1	12.5664 (0%)	12.5494 (0.135%)	12.5664
2	13.5736 (0.005%)	13.5461 (0.197%)	13.5729
3	15.7085 (0.013%)	15.6568 (0.316%)	15.7064
4	17.8574 (0.023%)	17.7729 (0.450%)	17.8533
5	25.1327 (0.0004%)	24.9999 (0.529%)	25.1328

3.2. Example 2

Figure 3 shows an axially loaded two-span beam at various boundary conditions: (*a*) clamped-free, (*b*) pinned-pinned, (*c*) clamped-pinned, and (*d*) clamped-clamped. Beam spans are of different lengths, with the first span having a length *aL* and the second (1-*a*)*L*, and it holds that 0<*a*<1.

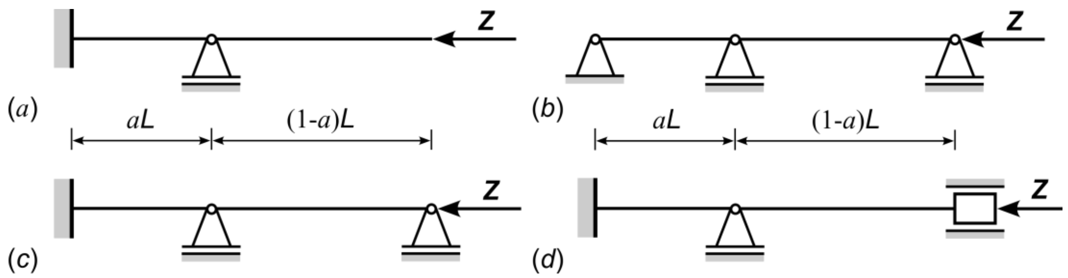

Figure 3: Beam with two spans: (*a*) C-F, (*b*) P-P, (*c*) C-P, (*d*) C-C.

The exact values of critical buckling force for various parameters under different boundary conditions are obtained in closed form in the reference [6]. By comparing the values of dimensionless critical buckling force (ZcrL2/(EIx))1/2 obtained by MSRM (with n=20 and 40) and the exact ones [6], both shown in Table 2, it is obvious that there is an excellent agreement. The relative errors between these two values which are shown in parenthesis, proven the convergence of the MSRM results to the exact ones as the number of used elastic segments n increase. Moreover, in some situations, using MSRM achieves the exact value, as is the case when a=0.7 at C-F and a=0.1 and 0.9 at the P-P boundary condition.

Table 2: The critical buckling load Z_{cr} of the beam with two spans at various boundary conditions (BC)

BC	*a*	MSRM		Exact [6]
		n=20	*n*=40	
C-F	0.1	1.7002 (0.124%)	1.6986 (0.029%)	1.6981
	0.3	2.0267 (0.039%)	2.0261 (0.01%)	2.0259
	0.5	2.5036 (0.02%)	2.5033 (0.008%)	2.5031
	0.7	3.2231 (0%)	3.2231 (0%)	3.2231
	0.9	4.1502 (0.031%)	4.1512 (0.007%)	4.1515
C-P	0.1	4.8640 (0.066%)	4.8616 (0.016%)	4.8608
	0.3	5.8248 (0.043%)	5.8267 (0.01%)	5.8273
	0.5	7.1416 (0.113%)	7.1477 (0.028%)	7.1497
	0.7	7.6153 (0.143%)	7.6235 (0.035%)	7.6262

BC	*a*	MSRM		Exact [6]
		n=20	*n*=40	
C-C	0.1	6.7920 (0.007%)	6.7916 (0.001%)	6.7915
	0.3	8.0247 (0.126%)	8.0323 (0.031%)	8.0348
	0.5	8.9698 (0.189%)	8.9826 (0.047%)	8.9868
	0.7	8.0247 (0.126%)	8.0323 (0.031%)	8.0348
	0.9	6.7920 (0.007%)	6.7916 (0.001%)	6.7915
P-P	0.1	4.8190 (0.004%)	4.8192 (0%)	4.8192
	0.3	5.6314 (0.067%)	5.6342 (0.018%)	5.6352
	0.5	6.2767 (0.103%)	6.2816 (0.025%)	6.2832
	0.7	5.6314 (0.067%)	5.6342 (0.018%)	5.6352

	0.9	6.7242 (0.065%)	6.7275 (0.016%)	6.7286		0.9	4.8190 (0.004%)	4.8192 (0%)	4.8192

3.3. Example 3

Figure 4 shows the three-span pinned-pinned beam of total length L=3m, which is loaded by the unit continuous load q=15 kN/m along the first span, force coupling M=5 kNm acting in the joint between the second and third span, as well as with the transversal force F=5 kN acting at the middle of the third span.

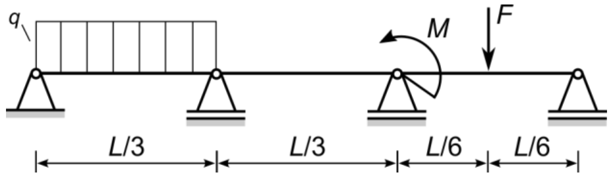

Figure 4: Loaded P-P beam with three spans

The coordinates of the deflection line are obtained in Table 3. The results obtained by using the MSRM and FEM are compared here. Note that n=150 elastic segments were used here in both methods. Looking at the relative errors of the MSRM versus FEM results, shown in parentheses, one can see the excellent agreement of the results. A small exception is a displacement at the point z=1.2 m, where a difference of 4.23% is observed. The assumption is that this difference occurs due to the small values of beam deformation at this point. The deflection line of the considered beam obtained by the proposed MSRM is shown in Figure 5.

Table 3: The coordinates of the deflection line of the loaded P-P beam with three spans

z_k [m]	$y_k \times 10^4$ [m]		z_k [m]	$y_k \times 10^4$ [m]	
	MSRM	FEM		MSRM	FEM
0.2	-235.968 (0.87 %)	-238.04	1.6	-245.043 (0.23 %)	- 244.49
0.4	-352.676 (0.93 %)	-355.97	1.8	-238.594 (0.12 %)	- 238.31
0.6	-308.826 (1.02 %)	-312.02	2.2	231.857 (0.13 %)	231.55
0.8	-148.269 (1.25 %)	-150.14	2.4	239.708 (0.23 %)	239.16
1.2	-12.8985 (4.23 %)	-12.375	2.6	165.85 (0.32%)	165.32
1.4	-132.196 (0.51 %)	-131.53	2.8	84.1399 (0.32 %)	83.868

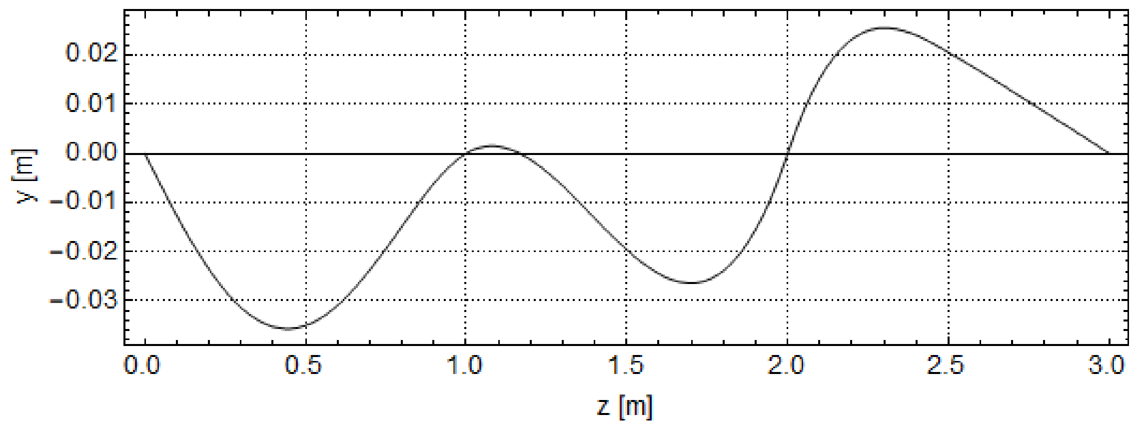

Figure 5: Deflection line of the loaded P-P beam with three spans

4. CONCLUSIONS

It is shown in the paper that the MSRM method developed in reference [7] for the static and dynamic analysis in the case of free-free beams can be easily modified for use in the analysis of multi-span beams. The use of absolute coordinates of rigid segments enables the easy introduction of different boundary conditions and static point loads into the analysis. The efficiency of the proposed method is shown through the three numerical examples, in which the obtained results are compared with those available in the literature. The relative error between these two groups of results is negligibly small and tends to zero with the increase in the number of used elastic segments. The proposed method has the potential for further use in the analysis of different types of beam-like structures composed of Euler-Bernoulli beams, considering its simplicity and achieved accuracy.

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ORIGINAL SCIENTIFIC PAPER OPEN ACCESS

Determination of relative weight coefficients based on thermal characteristics of materials using the F-AHP method

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ABSTRACT

In this paper, multi-criteria problem solving method – Fuzzy Analytical Hierarchy Process Method (F-AHP) is proposed. the goal of this paper is to implement the F-AHP procedure, which is to obtain weight coefficients based on the thermal characteristics of materials deposited in one warehouse. In subsequent research, the mentioned coefficients will be used in fire simulations inside the warehouse.

KEYWORDS

Multi-criteria, Fuzzy AHP method, Weight coefficients, Warehouse, Fire

1. INTRODUCTION

Effective management of product stocks in the warehouse is a key element in the work of any manufacturing or distribution company. In this regard, the classification of stored products is extremely important for the functioning of the warehouses themselves due to the large number of products that are deposited in the warehouse itself and the possibility of certain accidents (eg fire). Although warehouses do not have a high risk of fire, the danger is posed by warehouses of products such as explosives, fuel, solvents and other dangerous and explosive substances.

The analysis and modeling of fires in storage facilities, as well as the assessment of the risk of accidents, is attracting more and more attention from both researchers and engineers who design these facilities and systems. Fires in such buildings are characterized as very intense due to the high content of combustible matter per unit area in relation to the volume of the building. Depending on the type of warehouse and the fuel in the warehouse, in the event of a fire, smoke and toxic combustion products (containing carcinogenic substances) are emitted intensively, which contributes to harming the health of people inside the facility as well as air pollution.

Due to the storage of products and raw materials with different physical, mechanical and thermal characteristics, due to the creation of mathematical models and simulations, relative weight coefficients are necessary, which take into account all the listed material parameters.

The paper [1] presents a procedure for obtaining weight coefficients related to the overall functioning of a construction site as a system. Making the right decisions as one of the key prerequisites for achieving the planned goals in connection with the construction of facilities of different configurations. The realization of construction works itself sometimes represents a long process that requires the engagement of a lot of resources, either because of new projects, because of works that are in progress, or because of works that need to be realized in the near future.

Multi-criteria decision-making also plays a role in the selection of certain (specific conditions) related to the location of certain facilities, specifically a fire station in a seaport in Iran [2]. In this case, the ranking of criteria weights was performed using the AHP method, while the TOPSIS method was used in the ranking of alternatives and determining the best location for building the station itself.

Using the AHP method, air transport safety was assessed due to the fact that it is increasingly being developed and produced by new products, as well as goods that can be considered dangerous [3]. Due to the complexity of the problem itself and the number of parameters that were taken into account, the RFAHP method (Revised Fuzzy Analytical Hierarchy Process) was used in the paper.

After the performed procedures and calculations, results were obtained that can be further used in making decisions to improve performance in freight traffic. For this reason, more and more attention is being paid to the construction of hazardous materials warehouses, which locationally must meet certain criteria regarding the distance from urban settlements, roads, railways, as well as the distance from watercourses and water sources in order to avoid their potential pollution [4]. In order to solve this problem, a mathematical model was created, the AHP procedure was carried out, and finally, with the help of the GIS method, the locations of potential locations intended for the storage of hazardous materials were obtained.

Fires on electrical installations represent an important category due to the very characteristics of the mentioned installations. Fires on electrical installations in high-rise buildings are a big problem, and reliability and objectivity in risk assessment cannot be guaranteed for such buildings [5]. The first step in the research of the aforementioned fires is proposed, which is a numerical simulation to obtain data on the risk of fire. Then, using the AHP method, the ranking is performed and the weight coefficients of the fire risk assessment are obtained.

2. METHODOLOGY OF FUZZY AHP

Analytical Hierarchy Process (AHP) is a decision support method in order to prioritize among different criteria. In the process of obtaining the priority criteria, this method uses a comparison of the criteria paired with the measurement scale that has been determined. The main input parameters of the AHP method are the perception of the expert or experts, so that in the process of determining the relative weights, the factor of subjectivity plays a crucial role. The aspiration is to express the subjective opinion on the weights of the criteria by comparing the importance of the criteria in pairs at each hierarchical level due to significant uncertainty and the doubt that an inadequate assessment will affect the accuracy of the data and the results obtained. All the mentioned facts make the whole process of comparison too complicated, so in order to eliminate this shortcoming, fuzzy logic is used during the comparison, i.e. the Fuzzy Analytical Hierarchy Process (F-AHP) method is used. Due to their simplicity, when formulating a mathematical model for many problems, triangular and trapezoidal fuzzy numbers are most often used. The F-AHP method is used similarly to the AHP method, except that in the modified method, the AHP scale is set in the form of a fuzzy triangle that needs to be accessed (Figure 1).

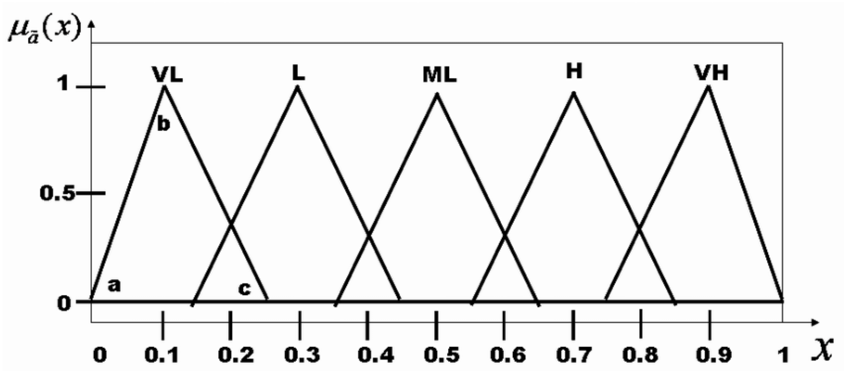

Figure 1: Triangular fuzzy number

In order to determine the priority of the criteria, i.e. importance of each criterion in relation to the main objective, the AHP method uses pairwise comparisons. For this purpose, a comparison matrix (an n×n square matrix) is formed in which each criterion in a row is compared with each criterion in a column at each level of the hierarchical structure. Saaty's scale of relative importance is used for comparison (table 3.1). Numerical values range from 1-9 and are divided into 5 basic degrees of intensity (1, 3, 5, 7 and 9) and 4 intermediate degrees (2, 4, 6 and 8).

In order to determine the priority of the criteria, i.e. importance of each criterion in relation to the main objective, the AHP method uses pairwise comparisons. For this purpose, a comparison matrix (square matrix) is formed in which each criterion in the row is compared with each criterion in the column at each level of the hierarchical structure. Saaty's scale of relative importance is used for comparison (Table 1). Numerical values range from 1-9 and are divided into 5 basic degrees of intensity (1, 3, 5, 7 and 9) and 4 intermediate degrees (2, 4, 6 and 8).

Table 1: Saaty's scale of significance (importance)

Numerical value	Definition	Reciprocal value
1	Equal importance	1
2	Weak	1/2
3	Moderate significance	1/3
4	Moderate plus	1/4
5	Strong significance	1/5
6	Strong plus	1/6
7	Very strong or proven significance	1/7
8	Very, very strong	1/8
9	Extreme significance	1/9

Let it be $C_1,...,C_m$ denoted by m decision criteria and let it be $W=(w_1,...,w_m)^T$ normalized relative importance weight vector, which is obtained by comparing pairs and satisfies the normalization condition $\sum_{j=1}^{m} w_j = 1$ with $w_j \geq 0$ for $j=1,...,m$.

Based on the results of the comparison of the criteria, a pair comparison matrix is formed $A(m \times n)$ shown in expression (1).

$$A=\left(a_{ij}\right)_{m\times n}=\begin{array}{c} C_1 \\ C_2 \\ \vdots \\ C_m \end{array}\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{nm} \end{bmatrix} \tag{1}$$

where a_{ij} represents a quantified judgment on w_i / w_j with $a_{ii}=1$ and $a_{ij}=1/a_{ii}$ for $i,j=1,...,m$. In case that pairwise comparision matrix $A=\left(a_{ij}\right)_{m\times n}$ satisfied that it is $a_{ij}=a_{ik}a_{kj}$ for any $i,j,k=1,...,m$, then it is claimed that A is a perfectly consistent matrix, otherwise it is inconsistent.

To determine the value of the vector of weighting coefficients $\{w_i\}$, several methods can be used from the matrix A, which essentially represent close approximations of the corresponding matrix elements A. Procedure for determining the value of a vector $\{w_i\}$ is to first determine the maximum eigenvalue of the matrix A – $\lambda_{\max}$, and in the second step determine the corresponding vector of eigenvalues that can be taken as a vector of approximate values of the weighting coefficients $\{w_i\}$, because (2):

$$\begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & \cdots \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & \cdots \frac{w_2}{w_n} \\ \cdots & \cdots & \cdots & \cdots \cdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \cdots \frac{w_n}{w_n} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{bmatrix}=n\begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{bmatrix} \tag{2}$$

Based on the criteria comparison matrix A, vector of weight coefficients $\{w_i\}$ can be determined by solving the characteristic equation (3):

$$AW=\lambda_{\max}W \tag{3}$$

where the mentioned method of determining the weight vector of the pairwise comparison matrix is called the right main eigenvector method.

It is assumed that the decision maker may not be able to provide ideally consistent pairwise solutions and the possibility of judgment errors exists, it is necessary that the pairwise comparison matrix A must have the required consistency which can be checked by the consistency ratio CR s shown in the following equation (4):

$$CR = \frac{(\lambda_{max} - n)/(n-1)}{RI} \tag{4}$$

where RI is random consistency index that depends on the row of the matrix, and the values are taken from Table 2, in which the first row represents the row of the matrix, and the second random index.

Table 2: Random consensus indicators.

n	1	2	3	4	5	6
RI	0	0	0.52	0.89	1.12	1.26

If it is $CR \leq 0{,}1$, the pairwise comparison matrix is considered to have acceptable consistency; otherwise, if the condition of consistency is not met, it is necessary to re-examine (revision) the comparison of pairs of criteria.

3. CASE STUDY - THE CALCULATION RESULT OF THERMAL CHARACTERISTICS OF MATERIALS BY F-AHP

The first step in the calculation of relative weight factors is the identification and selection of parameters that will be used in the creation and calculation of the weight index. The parameters taken into account as criteria are Ignition temperature, heat release rate (HRR), specific heat capacity and toxicity. The values of the mentioned parameters are taken from the relevant literature [6] are shown in Table 3.

Table 3: Values of parameters taken into account for index calculation

	Ignition temperature [°C]	Heat release rate HRR [MJ/kg]	Specific heat capacity [J/(kg K)]	Toxicity
Wood	250-300	17	1360	Normal
PVC plastic	100 - 260	20	880	Very high
Vulcanized Rubber	260-320	27	1880	Very high

The second step in the development of the proposed index is the creation of a procedure for determining the weight of each parameter, that is, the property taken into account. The first step in the implementation of the Analytical Hierarchical Process is already described in the previous step, which is the development of a hierarchical structure that will be applied in solving the problem (Figure 2). To obtain Weighting Coefficients at all levels of the hierarchy, it is necessary to perform a pairwise comparison of all elements of each subgroup [7].

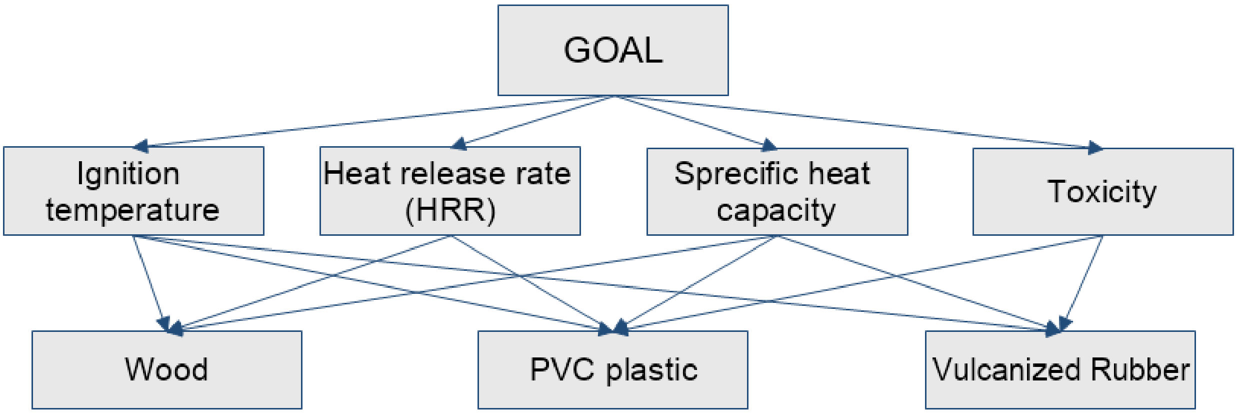

Figure 2: View of the Hierarchical structure taken into account [8]

In order to determine the relative importance of all pairs of criteria with the fulfillment of all previously mentioned rules and procedures, a pairwise comparison matrix was formed using Satie's scale of relative importance, which is shown in Table 4.

Table 4: Pairwise comparison matrix

	Ignition temperature [°C]	Heat release rate HRR [MJ/kg]	Specific heat capacity [J/(kgK)]	Toxicity
Ignition temperature [°C]	1	4 5 6	4 5 6	2 3 4
Heat release rate HRR [MJ/kg]	1/6 1/5 1/4	1	3 4 5	6 7 8
Specific heat capacity [J/(kg K)]	1/6 1/5 1/4	1/5 1/4 1/3	1	2 3 4
Toxicity	1/4 1/3 1/2	1/8 1/7 1/6	1/4 1/3 1/2	1

After the procedures were carried out, the individual results of each alternative for each criterion were obtained, which are shown in table 5. Based on the obtained data from the table, it is clearly seen that alternative A3, i.e. vulcanized rubber material has the highest weight coefficient.

Table 5: Summary results for each alternative according to each criterion

CRITERIA		Scores of Alternatives with respect to related Criterion		
	Normalized Weights (Ni)	**A1**	**A2**	**A3**
C1. Ignition temperature	0.536	0.223	0.325	0.452
C2. Heat release rate HRR	0.281	0.597	0.202	0.202
C3. Specific heat capacity	0.115	0.151	0.424	0.424
C4. Toxicity	0.068	0.597	0.202	0.202
TOTAL		**0.345**	**0.294**	**0.362**

4. CONCLUSIONS

Warehouses represent an important link in the supply chain. Warehouses that function efficiently allow the companies that own them to smoothly perform all the necessary operations within the warehouse itself. For the aforementioned reasons, a multi-criteria analysis using the F-AHP method was carried out in this work, taking into account several criteria in the form of thermal characteristics of materials such as ignition temperature, Heat release rate (HRR), calorific value, specific heat and toxicity. Toxicity was taken as one of the criteria due to the fact that due to the emission of harmful gases that occurs during combustion (CO, CO2, etc.) there is a possibility of harming the health of employees in the warehouse in the form of suffocation and ingestion of harmful particles. The materials considered as alternatives in this procedure are wood, PVC plastic and rubber. The paper also shows that the F-AHP method is simple, easy to use, and can be applied to a wide range of decision alternatives. The relative weight factors obtained using the mentioned method will represent the input parameters for further simulations and mathematical models for fires in warehouses and storage systems. In future research, it is possible to compare the results of several multi-criteria methods and determine which of the methods provides more accurate data on the weight coefficients.

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ORIGINAL SCIENTIFIC PAPER OPEN ACCESS

Evaluating the operation performance of the Serbian transport freight system by using Multiple Criteria Decision-Making technique

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ABSTRACT

Freight transport is a composite dynamic system that forms the unity of the infrastructure, vehicles, and personnel and which has its purpose or aim. Evaluating operation performance for freight transport systems is important for the government, companies, and users. This paper will use one of the most famous Multiple Criteria Decision-Making (MCDM) techniques to assess the operational performance of freight transport systems. At beginning, the authors will create the evaluation indicator system based on official data with four basic indicators and two sub-indicators for freight transport. These operational data/indicators represent the input of the chosen MCDM method. After that, the Analytic Hierarchy Process (AHP) method will use to calculate the indicator and sub-indicators weight and also will use to calculate the universal evaluate values and rankings of each transport mode performance. The Serbian freight transport system for the 2021st year will be selected as a case study for testing the AHP method within the MCDM approach; the associated suggestions to freight transport will also be given.

KEYWORDS

Freight transport, Operation performance, MCDM, AHP

1. INTRODUCTION

Every change that takes place in the economy and production of a country affects all segments of the development of the country as a business entity. If an economic entity does not have a certain flexibility and ability to adequately respond to changes that occur and affect its development, stagnation or cessation of economic growth of that entity occurs. Such events affect the development of all transport modes both in the world and in the Republic of Serbia. Although the Republic of Serbia has an extremely favorable geographical position as a transit country, transport has a very important role in the constant growth of the economic development of our country. Trade internationalization increases the demand for modern freight services. Freight transport is a vital in the supply chain industry due to increasing concerns about the environmental impact [1, 2].

As the development of our country has been noticeable in the last few years, especially in the development of road infrastructure, the challenges faced by freight transport are different. Today, freight transport users expect better service, fewer delays, and congestion, less time lost at border crossings, etc. Performance measurement allows the service provider to compare the goals they set and the results they managed to achieve. According to the White Book [3], performance optimization implies greater use of more energy-efficient transport modes. The 30% of road freight transport over 300 km should be directed to other modes such as rail and water transport by 2030, and more than 50% by 2050, facilitated by efficient and green freight corridors. To fulfill this goal, the development of appropriate infrastructure is also required. By 2050, it is necessary to connect all airports with a railway network, preferably at high speeds; provide sufficient connectivity of all seaports with railways for freight transport, and where possible, with inland waterways [3].

In order to remain competitive and respond to the rapidly changing market, companies must increase flexibility and adaptability in today's world market. In this sense, based on the measurement of operational performance, it is possible to choose the appropriate transport mode and it represents a multi-criteria problem that includes quantitative and qualitative criteria. The best way of monitoring the performance of some organization or some process is by the properly defined indicator or the set of indicators that are more specifically aimed at the observation area of interest. This way the management could monitor the entire system and make proper decisions to enhance their operations. One of the examples of measuring performance is a key performance indicator (KPI). Actually, a KPI is a universal tool that represents a measurable value that shows the effectiveness of a company in achieving its goals. Consequently, the KPI can be applied to measure the operational efficiency of transport modes from available data [4]. To create the total KPI of the transport freight system, many indicators and sub-indicators must be involved. Those indicators regard reliability, costs, flexibility and visibility, punctuality performance, mobility, capacity, business and financial performance, safety, etc. Decomposing a complicated and complex unstructured situation to the simpler components makes a hierarchical system problem the basis of the AHP method. The power of the AHP method consists in its ability to include all criteria and sub-criteria as relevant factors in the problem-solution process and as such it has found application in a various fields of economics, management, engineering, marketing, and education.

From a government perspective, evaluating and analyzing freight transport performance is essential because of the basis for government subsidies. Differences in performance between different transport modes are a strong justification for government financial subsidies. Also, the results provide support to the government on how to optimize and improve the structure of any transport mode. From the operator's perspective, freight transport performance evaluation can be seen as an efficiency assessment that focuses if operators achieving predetermined goals and what jobs they performed to achieve the goals. From the user's point of view, the evaluation of freight transport performance can be considered as time-saving accessibility and environmental friendliness.

After the introductory part, which discusses the importance and analysis of the operational performance of the freight transport modes, the second chapter is devoted to the review of the literature on the application of approaches and methodologies for evaluating the performance of the included freight transport modes. There is a wide spectrum of criteria that may be studied when it comes to the operational performance of freight rail, road, water (inland waterways), and air transport as a system, and for this reason, general settings for using multi-criteria decision-making and defining sets of input data are given in the third chapter. Furthermore, in the fourth chapter, an overview of the AHP method was given, which was used both for calculating weight coefficients and for ranking the performance of freight transport in the Republic of Serbia in 2021 based on valid statistical data. At the end of the research paper, the main conclusions and an overview of future studies tasks will be presented.

2. LITERATURE REVIEW

A review of the literature on freight transport modes shows that there are a limited number of studies that use different methods for decision-making based on multiple criteria to evaluate operational performance [5-7]. There are several approaches to measuring and evaluating freight transport performance. On the one hand, parametric approaches are rarely used because they need certain assumptions for determination the desired function; while on the other hand, researchers prefer to use nonparametric approaches which involve fewer assumptions [8].

Based on searches in databases, various MCDM techniques [9, 10] were used for measurement, evaluation and analysis performance in transport, such as Analytic Hierarchy Process - AHP, Analytic Network Process - ANP, VIše Kriterijumska Optimizacija Kompromisno Rešenje - VIKOR, Technique for Order of Preference by Similarity to Ideal Solution - TOPSIS, Decision making trial and evaluation laboratory - DEMATEL, Simple Additive Weighting - SAW and etc.

After a detailed review of MCDM methods, the paper [11] used the AHP method for a logistics company in choosing the most appropriate transport mode between the two listed locations in Turkey. The criteria that were used when choosing the transport mode are price, environmental acceptability, speed, safety, accessibility, flexibility, and reliability. Several price parameters (storage, transport, handling, crossing the Bosphorus) are included in the decision-making analysis. Research results indicated that rail transport, which is not widespread in Turkey, is the most convenient transport mode.

The paper [12] describes the model which uses AHP method for choosing a convenient transport mode from a potential three transport modes. This research aims to create a useful decision support tool for choosing the transport mode within the distribution logistics of motor fuels. The tool could help companies make the right decision on the choice of transport mode, taking into account the importance of different criteria that influence the decision-making analysis and process.

In paper [13], based on the AHP-TOPSIS combination method, a system for efficiently evaluating the logistics of maritime transport was constructed with port efficiency, port throughput, the efficiency of sea vessels, and ocean port cooperation as the indicators, and it was applied to the analysis of five chosen ports in China. The obtained results show that the port of Shanghai has the greatest efficiency in maritime transport.

Also, the use of the Data Envelopment Analysis - DEA method can be found in the evaluation of the performance of different transport modes. This method was used alone or in combination with MCDM methods. Yu (2008) conducted a study of efficiency and effectiveness using the DEA method for a group of forty large railway systems (freight and passenger) in 2002 [14]. In the paper [15], the authors used the DEA method to evaluate and analyze the efficiency of European railway companies taking into consideration different input and output configurations. Although companies from Western Europe showed higher performance that is, better results than companies from Central and Eastern Europe in passenger and overall transport, but it was not the case in freight transport.

The aim of paper [16] is to create, select, and evaluate the criteria that influence the efficiency of railway companies, increase their competitiveness and propose an approach based on DEA to assess the efficiency of railway companies. In order to solve the problem of criteria selection, the Fuzzy Analytical Hierarchical Processes - FAHP method was investigated, which demonstrated the priority of evaluating the railway companies' efficiency, with five groups of criteria. Criteria in a group that surpassed relative to other criteria for their freight railway companies within the adequate range were used as input and output indicators. The estimation of the efficiency of railway companies was investigated by using the DEA method. The obtained results show that the proposed method successfully enables the merging of a set of criteria (resource, safety, financial, quality, and operational) into a single estimation of the efficiency of railway companies until providing information on correctional measures that can improve the efficiency of railway companies.

In the paper [17], the authors presented a case study that examines different freight transport modes by a Turkish logistics service company. In practice, numerous conflicting qualitative and quantitative criteria can be used to evaluate alternative transport modes. Therefore, to solve the problem of ambiguity and uncertainty, ANP method in fuzzy environment was used. In essence, it is a basic form of the Analytic Hierarchy Process approach [18]. Many numbers of criteria that interconnect with each other are assessed and synthesized to select the most suitable transport mode. This assessment was conduct by a group of decision-makers (DM) coming from various levels of management in the logistics area and from the service company to ensure a reasonable solution. Moreover, the used model is confirmed by comparing the obtained results with the actual settings of the company.

Also, the urban railway transport system performance evaluation in the Chinese city of Chengdu for 34 months was performed using the Entropy - TOPSIS method [19]. The authors created a set of evaluation indicators with eight indicators and a total of fourty-one sub-indicators. The operational data of fourty-one sub-indicators were used as an input data for access.

Based on the review of the literature and indicated models and methods, and with the aim of determining the operational performance, a multi-criteria analysis will be conducted for freight transport system of the Republic of Serbia in 2021.

3. MCDM METHODOLOGY AND INPUT DATA

Multi-Criteria Decision Making (MCDM) is a subfield of Operational Research that was developed as a mathematical tool from both research and a practical perspective [20, 21]. MCDM methodology considers situations in which the DM must select only one alternative from a set of available alternatives that are evaluated based to more criteria. In order to make a good decision, it is necessary to define alternatives by forming appropriate criteria. As well, it is required to define the weight coefficients for every criterion, that is, the significance of every each in relation to the others criterion. Additionally, for each criterion, it is determined and defined whether it is necessary to select an alternative so that the criterion is minimal or maximal, i.e., what is the nature of each criterion. After that, toward to each criterion, the alternatives are valued and evaluated separately based on precisely determined parameters or subjective assessment. Characteristic way of presenting a MCDM problem is in matrix form. Criteria are often defined as quantitative measures that can be used "to simply illustrate and communicate complex phenomena, including trends and progress over time." Collected data can be suitable for analysis by those involved in decision-making and thereby contribute to better decision-making.

The Analytic Hierarchy Process method has been significantly used to untangle hierarchical decision-making problems in technical-technological areas. The theoretical basis of the AHP method is based on the following axioms [22]:

1. Axiom of reciprocity (if element X is n times more important than element Y, then Y is 1/n times more important than element X).

2. Axiom of homogeneity (a comparison only makes sense if the elements of sets are comparable, that is, element X can't be infinitely more significant than element Y).

3. Axiom of dependency (a hierarchy is formed, and a comparison is made between elements of one level of the hierarchy in relation to one element located at a higher level of the hierarchy. Lower-level comparisons do not affect an element of a higher-level).

4. Axiom of expectations (all elements of the hierarchy must be included in consideration and any change in the structure of the hierarchy requires a recalculation of the priorities in the new hierarchy. This practically means that the AHP method is sensitive to a subsequent change in the structure of the hierarchy).

Axioms 1 and 2 talk about evaluating pairs of hierarchy elements and assigning preferences, while axioms 3 and 4 describe formulating and solving problems as a hierarchy [22].

The AHP method is an effective tool that enables the complicated problem of MCDM to be created as a hierarchical decision-making structure. In addition, complex decision-making is simplified through a series of pairwise comparisons of hierarchy elements. After that, the results are synthesized and the significance coefficients of all elements of the hierarchy are determined. AHP also includes a technique and methodology for checking the consistency of decision-makers' ratings, which reduces affinity in the decision-making process. Method considers the set of evaluation criteria, and the set of alternatives from which to choose the best. Then AHP generates significance for each criterion, accordingly for a criterion pairwise comparison by the DM. What is higher the significance, the criterion is more important. Then, for each criterion, AHP assigns importance to each alternative, also based on pairwise comparisons by the decision maker. What is higher the significance, the alternative is better compared to the adopted criterion. At the end, AHP method represents combinations the importance of alternatives and criteria, and thus determines a global outcome for each alternative, as well as their ranking [10, 23].

Long-term research has formulated several criteria that are significant for evaluating the operational performance of freight transport modes. In this way, the operational goals are directly connected with the performance indicators of transport processes, and indirectly with the strategic goals of organizations. Figure 1 shows the algorithm for evaluating the operational performance of freight transport modes using the AHP method. The fact is that in practice there is no uneque model that fits every situation.

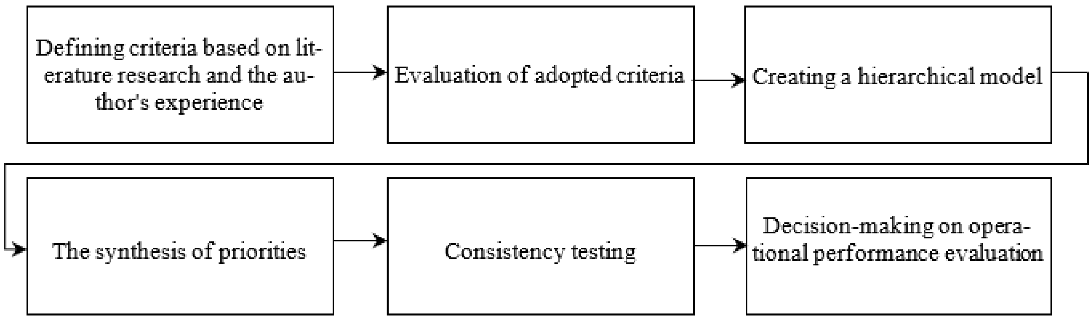

Figure 1: Algorithm for evaluating the operational performance of freight transport modes using the AHP method

In the conducted analysis for evaluating the operational performance of freight transport modes, the available data collected by statistical reports of transport companies consolidated in the Statistical Yearbook of the Republic of Serbia were used [24]. The data for goods transport by inland waterways and air transport refers to transport realized by transport organizations registered for transport activities, whether the transport was carried out within or outside of the Republic of Serbia. Operations and activity in railway transport, displayed in passenger kilometres and ton kilometres, refer to transport carried out within the territory or outside the borders of the Republic of Serbia. Domestic inland waterway freight transport includes the total transport activity at river ports.

Quantity matrix with transport modes as alternatives and indicators with sub-indicators as criteria are given in Table 1. The freight transport modes taken into consideration are V_1 - Rail, V_2 - Road, V_3 – Water (Inland Waterways), and V_4 - Air transport. It includes 4 indicators, of which one indicator is based on two sub-indicators, which are used to evaluate the freight transport system performance of the Republic of Serbia in 2021. The following indicators were used: K_1 - Carrying capacity of transport (Ton of carrying capacity, thous. t), K_2 - Number of employees in transport, K_3 - Consumption of fuel in transport (Liquid fuels, thousand tons) and K_4 - Basic indicators of transport. K_{41} - Achieved tkm (million) and K_{42} - Goods carried (thousand tons) were used as sub-indicators of the basic indicators.

Table 1: Quantity matrix

Transport modes	Indicators				
	Indicator K_1	Indicator K_2	Indicator K_3	Indicator K_4	
				Sub-indicator K_{41}	Sub-indicator K_{42}
Rail, V_1	243	10497	12	2925	10783
Road, V_2	248	23679	264	10108	23400
Water (IWW), V_3	381	1253	21	1883	6597
Air, V_4	0.30	2736	81	17.10	4730

4. THE APPROACH AND RESULTS DISCUSSION FOR CASE STUDY: THE SERBIAN FREIGHT TRANSPORT MODES

Economic globalization and the emergence of new consumption behaviors have led to the increase in volume of goods transported and the distance traveled. Measuring and analysis the performance of freight transport modes unavoidably becomes a precondition for their existence on the market in today's dynamic and very highly turbulent ambience. Carriers must bring and find optimal decisions and solutions for the purpose of efficient and effective business, overlive in the transport market, as well as to improve their competitive advantages.

To evaluate the operational performance of freight transport modes using the AHP method, a matrix of relevant importance criteria was formed (Table 2). By comparing the criteria according to Satie's scale [23], on the basis the literature and the author's experience, the members of the normed criteria importance matrix were determined.

The relative importance matrix of the criteria shows that the indicator K_1 - Carrying capacity of transport is more dominant compared to the indicator K_2 - Number of employees in transport, while in the case of indicators K_3 - Consumption of fuel in transport and K_4 - Basic indicators of transport (Achieved tkm and Goods carried) it has less importance. Indicator K_2 also has less dominance in relation to indicator K_3 and sub-indicators K_{41} and K_{42}. Indicator K_3 is more dominant in relation to sub-indicator K_{41}, while in relation to sub-indicator K_{42} it has the same importance. Sub-indicator K_{41} has the same importance in relation to sub-indicator K_{42}.

Table 2: Relative Importance Matrix of the criteria

	K_1	K_2	K_3	K_{41}	K_{42}
	max	*min*	*min*	*max*	*max*
K_1	1	2	0.5	0.2	0.2
K_2	0.5	1	0.14	0.2	0.2
K_3	2	7	1	2	1
K_{41}	5	5	0.5	1	1
K_{42}	5	5	1	1	1

As there are several different criteria, which do not have the same importance, weighting coefficients were determined, based on which their relative importance w_{ki} is obtained (Table 3). The weighting coefficients should show the importance of the participation of certain criteria in making a decision on the ranking of alternatives, i.e. transport modes.

Table 3: Operational performance evaluation values and rankings of for each transport mode

Indicator		Fact. of importance of the variants of transp. modes				Indicator of variants of types of transport			
Label K_i	Weight w_{ki}	Rail w_{rijV1}	Road w_{rijV2}	Water w_{rijV3}	Air w_{rijV4}	Rail u_{rijV1}	Road u_{rijV2}	Water u_{rijV3}	Air u_{rijV4}
K_1	0.09	0.249	0.298	0.448	0.006	0.021	0.0261	0.0391	0.0004
K_2	0.05	0.096	0.041	0.575	0.288	0.0045	0.0019	0.0272	0.0136
K_3	0.31	0.578	0.036	0.289	0.096	0.1810	0.0113	0.0905	0.0301
K_4	0.55	0.529	1.060	0.303	0.108	0.146	0.290	0.083	0.031
K_{41}	0.26	0.229	0.610	0.153	0.008	0.0593	0.1587	0.0396	0.0022
K_{42}	0.29	0.300	0.450	0.150	0.100	0.0875	0.1313	0.0437	0.0291
Total indicator of the variants of transport modes $U_{rik} = \sum u_{rijk} \ \forall k = V_1, V_2, V_3, V_4$						**0.3544**	**0.3295**	**0.2403**	**0.0756**

The conducted analysis (Table 3) shows that indicator K_4 - Basic indicators of transport have the greatest influence in the evaluation system (w_{k4}=0.55), while K_{41} - Achieved tkm have less importance (w_{k41}=0.26) compared to K_{42} - Goods carried (w_{k42}=0.29). Indicators K_1 - Carrying capacity of transport and K_2 - Number of employees in transport have the lowest values of weight coefficients, respectively (w_{k1}=0.09) and (w_{k2}=0.05) and show that they have the least impact on the evaluation process of operational performance.

Further analysis leads to the significance factor of the variants of the transport modes w_{rijVi} (Table 3) by comparing each indicator with each variant of the transport modes. Indicators of variants of transport modes according to each indicator (sub-indicator) are determined by the equation:

$$\mathrm{U}_{\mathrm{rijVi}} = w_{ki} \cdot w_{rijVi} \tag{1}$$

Based on the indicators of the variants of transport modes according to each indicator, the total indicators for each transport mode were determined:

$$\mathrm{U}_{\mathrm{rik}} = \sum u_{rijk} \quad \forall k = V_1, V_2, V_3, V_4 \tag{2}$$

Based on the ranking performed by the AHP method (Table 3), based on the adopted indicators for freight transport in the Republic of Serbia in 2021, it can be concluded that the best business results were achieved by railway freight transport (u_{rijV1} =0.3544), road transport (u_{rijV2} =0.3295) in second place, followed by inland waterway transport (u_{rijV3}=0.2403), while air freight transport recorded the worst scenario (u_{rijV4}=0.0756).

The trend of railway development both in developed countries and in the Republic of Serbia is moving towards its modernization, improving capacity, and increasing the throughput capacity of railways, generally by improving exploitation and transport organization. In addition to the uneven development of railways in individual regions and countries, another characteristic of the development of railway transport is the unevenness in the development of transport capacities. It is reflected in the better equipment of the railways, more developed rolling stock, greater capacity, etc. Today, the railways have lost much of their power and left the role of the main transporter to road transport. Air freight transport is irreplaceable on long distances, while on medium distances its comparative advantage is threatened by rail and road transport.

5. CONCLUSIONS

The latest trends in performance management imply an orientation towards results, which means not only managing effective behavior but also achieving high results. Transport is essential to our economy and society. New transport models must appear, by which larger cargo quantities are transported to their destination by the most efficient transport mode. It is preferable to use individual transport for final destinations. In order to successfully respond to the very clear requirements of the modern business environment, transport organizations must clearly define their determination in the direction of increasing the efficiency of functioning, i.e. accepting the logic of the market, applying modern management methods and constantly measuring the level of development achieved, which is the essence of the philosophy that leads the organization to success, that is, long-term sustainable business.

In the paper, a multi-criteria analysis using the AHP method was carried out in order to evaluate the operational performance of freight transport modes in the Republic of Serbia in 2021. The alternatives were Rail, Road, Water (Inland waterways), and Air transport, while the Carrying capacity of transport, Number of employees in transport, Consumption of fuel in transport as well as Basic indicators of transport - Achieved tkm and Goods carried were used as criteria or indicators. The best and worst operational performances were achieved respectively by Rail and Air freight transport.

The future development of freight transport must be based on:

- Energy efficiency improvement the of vehicle performance in all modes of transport. Development and application of renewable energy sources and propulsion systems.
- Optimizing the multimodal transport performance, which implies greater use of essentially more efficient transport modes, where other technological innovations may fall short.
- The more efficient transport and infrastructure use in application for the improved management of transport and modern information systems, progressive transport, logistics and market measures such as the full development and modernization of the integrated European railway market, the abolition of cabotage restrictions, the removal of barriers for short sea deliveries, undistorted prices, etc.

From everything mentioned so far, it can be deduced that the transport services market related to freight transport is extremely developed and that there is strong competition in certain transport modes. Freight shipments at short and medium distances (under some 300 km) will mostly stay on the trucks. Therefore, it is important, in addition to encouraging an alternative transport solution (rail, water transport), to improve the efficiency of trucks, using intelligent and modern transport systems and extra measures to advancement market mechanisms.

The framework of future research would be reflected in the MCDM approach to assessing the performance of other systems using compatible evaluation indicators.

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