

Metaheuristic Applications in Mechanical and Structural Design

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ARTICLE INFO

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DOI: 10.5937/engtoday2201019P

UDC: 621(497.11)

ISSN: 2812-9474

Article history: Received 24 February 2022; Accepted 30 March 2022

ABSTRACT

The paper shows the significance of metaheuristic optimization algorithms through their application to specific engineering problems, especially in mechanical and civil engineering domains, where some significant publications are presented. Moreover, due to their nature, these algorithms are very convenient for application in various engineering examples, both with single-objective or multi-objective optimization problems. Also, they are successfully being applied for tasks with a great number of variables and constraint functions. Finally, the paper presents the comparison of the results of seven chosen metaheuristic optimization algorithms that were applied on the example of the cantilever beam subjected to complex loading. The objective function was the cross-sectional area of the welded I-profile. In contrast, the constraint functions were the permissible stresses in the I-profile and the welded connection supporting a cantilever beam and one welding technology limitation. After comparing obtained optimum results, optimization time and convergence for all seven chosen algorithms, some conclusions and recommendations for an appropriate type choice and application were made.

KEYWORDS

Metaheuristic, Optimization, Bi-moment, Restrained warping, Cantilever beam, Mechanical design, Structural design

1. INTRODUCTION

Metaheuristic algorithms gain more and more appliances in optimization, especially in constrained engineering problems. Recent years have seen many bio-inspired metaheuristic algorithms emerge. Their application is frequent in various engineering problems, both with single-objective or multi-objective optimization problems, due to the possibility of managing many variables and constraint functions.

Metaheuristic algorithms are very efficient, and they don't require recalculations of derivatives of the objective function (which is required for the gradient optimization methods). The initial values of variables can be set within pretty wide bounds. The designer doesn't have to be experienced in setting the initial values of variables (which is very important). Metaheuristic algorithms can generate new solutions that are better in comparison to the previous ones and are very successfully avoiding the trap of entering a local optimum.

These algorithms are being applied in everyday research, so many research papers present their utilisation in the mechanical and civil engineering domain [1-20].

An improved new swarm intelligence optimization algorithm named the Multi-Specular Reflection Algorithm (M-SRA) is utilised in [1] to optimize the main girder of the double-beam bridge crane concerning lightweight and green design. The optimization of the main girder of the single-beam bridge crane is conducted in [2] by Bat Algorithm (BA), Cuckoo Search (CS) algorithm, and Firefly Algorithm (FA). Significant savings were achieved in relation to previous solutions, where the algorithm with the best results was revealed. Two algorithms inspired by laws of physics - Charged System Search (CSS) algorithm and Thermal Exchange Optimization (TEO) algorithm – were used in the optimization of cross-sections of the crane hook [3], where two cross-section types were considered. Again, significant savings were achieved in comparison with present solutions. Besides the mentioned mechanical engineering problems, these algorithms were successfully used for solving: gear train design problem [4-8], welded beam design problem [4, 5, 7-13], pressure vessel design problem [4, 5, 9, 11-13], tension/compression spring design problem [4, 7-13], cantilever beam design problem [4-6, 10], speed reducer design problem and rolling element bearing design problem [11, 12]. There was a comparison of applied algorithms and obtained results for each problem.

In addition, some of the metaheuristic algorithms are used in conjunction with Computational Fluid Dynamic (CFD) analysis for solving the airfoil design problem [10], using Salp Swarm Algorithm (SSA). In the same article, the mentioned algorithm was also used for multi-objective optimization in the marine propeller design problem. Furthermore, Sine Cosine Algorithm (SCA) was used for multi-objective optimization in the airfoil design problem [14]. The propeller design issue was considered as a single-objective optimization problem for efficiency increase [4, 6]. However, the submarine propeller design problem [15] was subject to multi-objective optimization by Dragonfly Algorithm (DA).

Besides mechanical design problems, these algorithms are frequently utilised in structural engineering. For example, [4, 10] considers the I-beam design problem, while [4-6, 10] studies the 3-bar truss design problem. In addition, the truss optimization problem has much research on it: 15-bar and 52-bar truss design problems were studied in [4, 9], 25-bar truss design problem (the base of transmission tower) was considered in [9, 11, 12, 16], and 72-bar truss design problem was addressed in [16, 17]. Again, there was a comparison of applied algorithms and obtained results for each problem.

Research [16] optimized the 120-bar dome truss structure and the planar 200-bar truss structure by Cyclical Parthenogenesis Algorithm (CPA). Also, the tower truss design problem (582-bar truss design problem) was solved by Enhanced Vibrating Particles System (EVPS) algorithm, [17]. The application of various technics and metaheuristic algorithms on many structural design problems and general engineering problems was presented in [18].

Authors in [19] presented the procedure of optimization for the steel girders in a high strength steel composite bridge from the viewpoint of weight, material cost and environmental impact. The Genetic Algorithm (GA) was used to solve this problem. Research [20] considered the problem of optimal design of buckling-restrained braced frames by using SSA and Enhanced Colliding Bodies Optimization (ECBO) algorithm.

So, the application of these algorithms is massive for various engineering problems. Besides showing these applications, especially in mechanical and civil engineering domains, this research aims to compare the results of more metaheuristic optimization algorithms applied to one specific engineering problem and give some directions about the choice of algorithm and its implementation.

2. APPLICATION OF METAHEURISTIC OPTIMIZATION ALGORITHMS

This research uses a welded I-section cantilever beam with fixed support [21] as an example of the engineering structure upon which the metaheuristic algorithms will be applied. The beam is subjected to torsion (M_z), axial eccentric force (F_z) and bending moment (M_{y1}), as depicted in Figure 1.

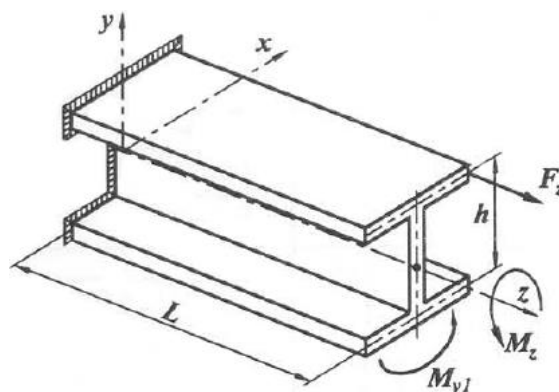


Figure 1: A cantilever beam

The goal of optimization is the reduction of the cross-sectional area of the welded I-profile. This single-objective multi-criteria optimization problem is a constrained optimization problem, and it is defined in the following way:

$$\min F(X) = f(X) + P(X, k_i) \tag{1}$$

$$P(X, k_i) = \sum_{i=1}^{n_g} k_i \cdot [\max(0, g_i(X))]^2 \tag{2}$$

where $f(X)$ is the objective function, X is the vector made of design variables, $P(X, k_i)$ are the penalty functions, k_i are the penalty factors, $g_i(X)$ are the constraint functions, and n_g is the total number of constraint functions.

Variables (x_j) are defined by their lower (l_j) and upper bounds (u_j). This research treats five variables (Figure 2).

The objective function and constraint functions are presented in the following chapters (Chapter 2.1 and Chapter 2.2).

2.1. The Objective Function

The cross-sectional area (A_p) of the welded I-profile (the cross-sectional area of plates) represents the objective function $f(X)$ in this optimization problem (Figure 2). The area of the welded connection (A_s) in support of a cantilever beam depends on this area (A_p).

The objective function is defined as follows:

$$f(X) = A_p = 2 \cdot b \cdot t + h \cdot d \tag{3}$$

All important geometric quantities are marked and defined, as shown in Figure 2.

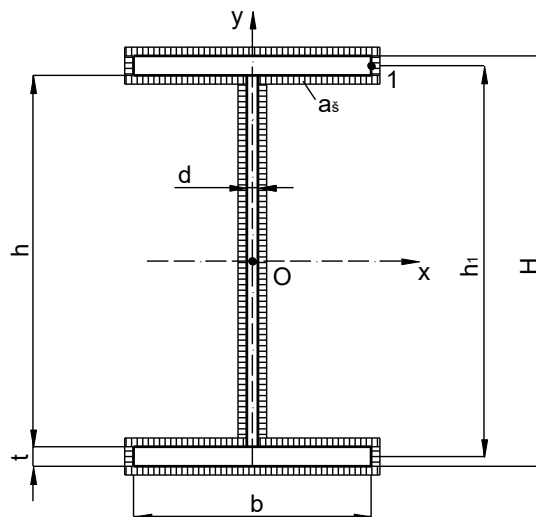


Figure 2: The cross-sectional area of the welded I-profile

The variables in this optimization process are:

$x_1 = b$ - the flange width, $x_2 = h$ - the web height, $x_3 = t$ - the flange thickness, $x_4 = d$ - the web thickness, and $x_5 = a_s$ - the weld thickness.

2.2. The Constraint Functions

In addition to the bending and tensile stresses that occur in the I-profile and the welded connection in support of a cantilever beam, stresses due to the restrained warping are also presented (Figure 1).

The maximum stresses due to the restrained warping in the I-profile and the welded connection in support of a cantilever beam, respectively:

$$\sigma_\omega = \left| \frac{B_\omega \cdot \omega_1}{I_\omega} \right| \tag{4}$$

$$\sigma_{\omega_s} = \frac{\sigma_{\omega_1} \cdot t}{2 \cdot a_s} \tag{5}$$

The quantities present in (4) and (5) are determined according to [21], where:

$$B_o = -E \cdot I_\omega \cdot C_o \cdot C_\omega \quad (6)$$

$$I_\omega = \frac{1}{24} \cdot t \cdot h_1^2 \cdot b^3 \quad (7)$$

$$C_o = \frac{M_{\omega t} \cdot \alpha}{G \cdot I_t} \quad (8)$$

$$\alpha = \sqrt{\frac{G \cdot I_t}{E \cdot I_\omega}} \quad (9)$$

$$G = \frac{E}{2 \cdot (1 + \nu)} \quad (10)$$

$$I_t = \frac{1}{3} \cdot (2 \cdot b \cdot t^3 + h \cdot d^3) \quad (11)$$

$$C_\omega = \operatorname{tgh}(\alpha \cdot L) - \frac{B_s \cdot \alpha}{M_{\omega t} \cdot \cosh(\alpha \cdot L)} \quad (12)$$

$$B_s = F_z \cdot \omega_1 - \frac{M_{y1} \cdot h_1}{2} \quad (13)$$

$$\omega_1 = -\frac{1}{4} \cdot b \cdot h_1 \quad (14)$$

where B_o and B_s are bi-moments, $M_{\omega t} = M_z$ is the torsion moment, I_ω is the sectorial moment of inertia, I_t is the torsional moment of inertia, α is the torsion parameter, G is the shear modulus of the plate, E is the elastic modulus of the plate, ν is Poisson's ratio of the plate, ω_1 is the sectorial coordinate at point 1, and C_o and C_ω are constants, [21].

The maximum stresses due to bending moments and tensile force in the I-profile and the welded connection in support of a cantilever beam, respectively:

$$\sigma_z = \frac{M_x}{W_x} - \frac{M_y}{W_y} + \frac{F_z}{A_p} \quad (15)$$

$$\sigma_{z_s} = \frac{M_x}{W_{x_s}} - \frac{M_y}{W_{y_s}} + \frac{F_z}{A_s} \quad (16)$$

where W_x and W_y are section moduli for the I-profile, W_{x_s} and W_{y_s} are section moduli for the welded connection in support of a cantilever beam (Figure 2), and M_x and M_y are bending moments in x and y direction (Figure 1), respectively:

$$M_x = \frac{F_z \cdot h_1}{2} \quad (17)$$

$$M_y = M_{y1} - \frac{F_z \cdot b}{2} \quad (18)$$

The total stresses in the I-profile (σ_u) and the welded connection in support of a cantilever beam (n_s), respectively:

$$\sigma_u = \sigma_\omega + \sigma_z \leq \sigma_d \quad (19)$$

$$n_s = \sigma_{\omega_s} + \sigma_{z_s} \leq \sigma_{s,d} \quad (20)$$

The permissible stresses in the I-profile (σ_d) and the welded connection in support of a cantilever beam ($\sigma_{s,d}$), respectively:

$$\sigma_d = \frac{R_e}{\nu_1} \quad (21)$$

$$\sigma_{s,d} = 0.75 \cdot \beta \cdot \sigma_d \quad (22)$$

where $\beta = 1$, for $a_s \leq 7 \text{ mm}$; $\beta = f(a_s)$, according to [21], for $a_s > 7 \text{ mm}$.

The constraint functions are defined based on previously determined stresses in the material of the I-section and the welds in the beam support and limitation imposed by welding technology [21]:

$$g_1 = \sigma_u - \sigma_d \leq 0 \quad (23)$$

$$g_2 = n_s - \sigma_{s,d} \leq 0 \quad (24)$$

$$g_3 = a_s - 0.7 \cdot \min(t, d) \leq 0 \quad (25)$$

2.3. Metaheuristic Algorithms

The optimization of the considered example was carried out by applying seven selected metaheuristic optimization algorithms. The following algorithms were applied: Moth-Flame Optimization (MFO) algorithm, Multi-Verse Optimizer (MVO), Ant Lion Optimizer (ALO), Whale Optimization Algorithm (WOA), Sine Cosine Algorithm (SCA), Dragonfly Algorithm (DA), and Locust Search (LS) algorithm, [4-6, 9, 14, 15, 22].

The Moth-Flame Optimization (MFO) algorithm is a population-based nature-inspired algorithm, based on the computer simulation of the navigation of moths, [4]. The Multi-Verse Optimizer (MVO) is a nature-inspired algorithm, based on three concepts in cosmology: white hole, black hole, and wormhole, [5]. The Ant Lion Optimizer (ALO) is an algorithm that mimics the hunting mechanism of antlions in nature, [6]. Similar to the previous, Whale Optimization Algorithm (WOA) mimics the social behaviour of humpback whales. This algorithm is inspired by the bubble-net hunting strategy, [9]. The Sine Cosine Algorithm (SCA) is a population-based algorithm that creates multiple initial random candidate solutions and requires them to fluctuate outwards or towards the best solution using the sine and cosine functions, [14]. The Dragonfly Algorithm (DA) is a swarm intelligence optimization method, inspired by the static and dynamic swarming behaviours of dragonflies in nature, [15]. The Locust Search (LS) algorithm is a nature-inspired algorithm, based on the behaviour of swarms of locusts, [22].

2.4. Results of Optimization

The application of the selected optimization algorithms was carried out on the considered example of a cantilever beam, where the following optimization parameters were adopted for all algorithms:

$N = 100$ - the population size; *Number of iteration* = 1000 - the maximum number of iterations.

The objective function are defined by (3). Constrained functions are defined by (23)-(25).

The bound values of variables are:

$$20 \leq x_1 \leq 30, 20 \leq x_2 \leq 80, 0.6 \leq x_3 \leq 4, 0.5 \leq x_4 \leq 3, a_{s,min} \leq x_5 \leq 1.2.$$

The minimum weld thickness, according to [21], is: $a_{s,min} = 3 \text{ mm}$.

The input parameters for the optimization process are (Figure 1):

$$L = 200 \text{ cm}, M_z = 300 \text{ kNcm}, M_{y1} = 350 \text{ kNcm}, F_z = 250 \text{ kN}, E = 21000 \frac{\text{kN}}{\text{cm}^2}, R_e = 35.5 \frac{\text{kN}}{\text{cm}^2}, \nu = 0.3, \nu_1 = 1.5,$$

where L is the length of a cantilever beam, M_z , M_{y1} , and F_z are loads that act on a cantilever beam, R_e is the minimum value for the yield stress, and ν_1 - is load case 1 factored load coefficient.

All algorithms were run several times, and the results from two simulations were chosen and given in the following tables and convergence diagrams (Table 1 and Table 2, Figure 3 and Figure 3).

Tables 1 and 2 show the optimization results for all seven metaheuristic algorithms applied to the considered example. Besides obtained optimized section areas (Best – the best value) and optimized variable values, each algorithm's characteristics are shown (Worst – the worst value, Mean – the mean value, Std – standard deviation, and Time - optimization run time).

There was no violation of the constraint functions in all applied algorithms within the optimization process execution (values g_1 , g_2 and g_3 , Table 1 and Table 2).

Table 1: Optimization results for Simulation 1

	MFO	ALO	DA	LS	WOA	SCA	MVO
Best [cm ²]	133.948	134.994	137.143	134.537	137.910	137.370	134.625
Worst [cm ²]	214.979	203.246	234.435	243.024	296.139	203.452	231.039

	MFO	ALO	DA	LS	WOA	SCA	MVO
Mean [cm ²]	134.578	135.649	138.267	135.661	141.898	139.439	137.850
Std	5.759	3.086	6.838	5.590	10.164	4.671	6.754
Time [s]	2.885	37.075	73.481	80.728	3.907	2.916	4.831
g ₁	-2.5221	-2.9998	-3.0209	-3.2411	-3.8660	-3.3827	-3.3591
g ₂	0	0	-0.3650	0	0	-0.2626	0
g ₃	0	0	0	0	-0.0411	-0.0007	0
x ₁ [cm]	29.0681	24.6512	30.0	22.7058	23.1879	25.0776	21.8516
x ₂ [cm]	20.0	20.0	20.0	20.0	20.0	20.0	20.0
x ₃ [cm]	1.7143	1.7143	1.7143	1.7147	1.7462	1.7152	1.7145
x ₄ [cm]	1.7143	2.5237	1.7143	2.8334	2.8463	2.5671	2.9849
x ₅ [cm]	1.2	1.2	1.2	1.2	1.1813	1.2	1.2

Table 2: Optimization results for Simulation 2

	MFO	ALO	DA	LS	WOA	SCA	MVO
Best [cm ²]	134.507	133.985	133.948	134.090	140.066	139.031	133.994
Worst [cm ²]	194.027	247.817	190.042	169.281	284.358	209.991	248.398
Mean [cm ²]	134.852	134.585	135.015	140.531	142.209	142.783	135.595
Std	3.160	4.219	5.046	5.644	7.273	5.628	5.185
Time [s]	4.044	46.826	115.944	100.556	4.918	4.096	5.415
g ₁	-3.2558	-2.5248	-2.5222	-2.5397	-3.2804	-3.5510	-2.5206
g ₂	0	0	0	-0.0102	0	-0.0679	0
g ₃	0	0	0	0	-0.0374	-0.0334	0
x ₁ [cm]	22.551	29.0474	29.0677	29.0657	29.1140	26.0471	29.1205
x ₂ [cm]	20.0	20.0	20.0	20.0	20.0	20.0	20.0
x ₃ [cm]	1.7143	1.7143	1.7143	1.7143	1.6965	1.7272	1.7126
x ₄ [cm]	2.8595	1.7197	1.7144	1.7215	2.0641	2.4527	1.7126
x ₅ [cm]	1.2	1.2	1.2	1.2	1.1502	1.1757	1.1988

For some algorithms, 100 iterations are enough to achieve an optimum solution (convergence diagrams in Figure 3 and Figure 4), while some algorithms need 500 or 1000 iterations, which means that the chosen number of iterations was chosen adequately to make the comparison in this case.

It is noticeable that the optimum value of variable $x_2 = h$ is its minimum (Table 1 and Table 2). On the opposite, the optimized value of variable $x_5 = a_5$ is at its upper bound (maximum) or close to it.

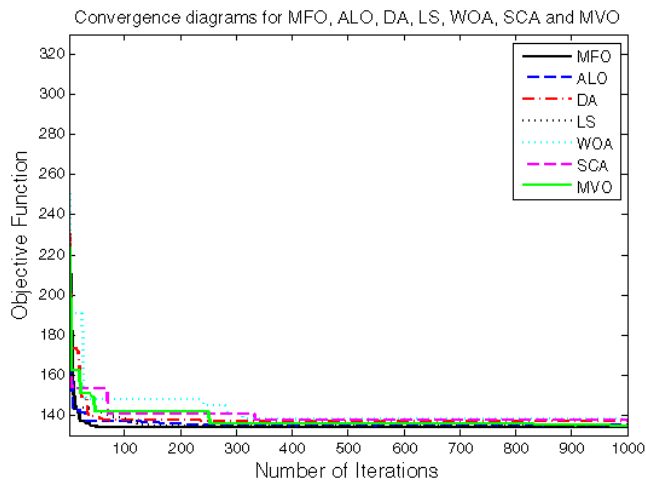


Figure 3: Convergence diagrams for Simulation 1

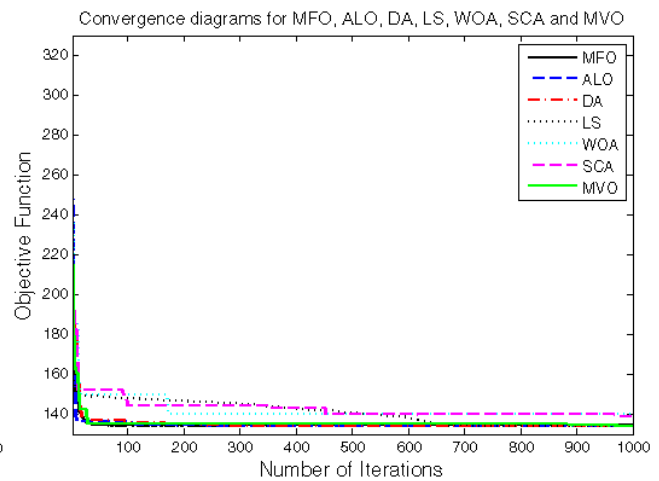


Figure 4: Convergence diagrams for Simulation 2

3. CONCLUSION

This research presents the importance of applying metaheuristic optimization algorithms to mechanical and structural design problems. Also, seven algorithms are applied to a single-objective engineering problem. The objective function is the cross-sectional area of the welded I-profile, shown in Figure 2. The optimization problem has five variables and three constraint functions. The results are taken out of two simulations per each applied algorithm.

For the considered case, input data and optimization parameters, it has been revealed that MFO reaches the optimum value for the shortest time (Table 1 and Table 2). Also, WOA, SCA, and MVO reach the optimum value quickly. However, all other algorithms need significantly longer to optimize, especially DA and LS algorithms.

By looking at the obtained optimum values and differences in constraint functions (g_1 , g_2 and g_3), the tables show that WOA and SCA algorithms yield the poorest results compared to other algorithms (Table 1 and Table 2, Figure 3 and Figure 4). Moreover, the WOA algorithm has the greatest standard deviation (Table 1 and Table 2).

MFO and MVO algorithms emerged as the most appropriate considering the minimum cross-sectional area values, standard deviation, and optimization time. However, the MFO algorithm might be modified to mitigate early convergence (Figure 3 and Figure 4), which would improve the accuracy. The ALO algorithm is also very suitable for the application in terms of accuracy and standard deviation but should be improved to shorten the processing time.

Further research needs new algorithms to be tested on specific engineering problems with many variables and constraint functions, both in single-objective and multi-objective optimization problems. Finally, the researchers should undertake modifications, hybridizations and improvements of algorithms and their parameters to improve accuracy, convergence and speed.

ACKNOWLEDGEMENTS

This work has been supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia, through the Contracts for the scientific research activity realisation and financing in 2022, 451-03-68/2022-14/200108 and 451-03-68/2022-14/200102.

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