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Determination of the distribution law for the time-to-failure of the high-pressure pump in the engine

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ABSTRACT

This paper investigates the reliability of high-pressure pumps in engines using statistical and theoretical models with the aim of optimizing their maintenance. The first part of the paper analyzes various distributions of time-to-failure for high-pressure pumps, including exponential, Weibull, and normal distributions, using the Kolmogorov-Smirnov test to evaluate the fit between empirical and theoretical distributions. Based on these analyses, the exponential distribution was identified as the most suitable approximative reliability model for high-pressure pumps. The second part of the paper focuses on determining the optimal maintenance period for high-pressure pumps based on the criterion of maximum readiness, as well as evaluating whether there is an optimal time for minimizing maintenance costs.

KEYWORDS

Reliability, Readiness, Minimal costs, Maintenance optimization, Reliability indicators, Reliability model, Statistical measures, Distribution approximation.

1. INTRODUCTION

This work is a continuation of the research on testing [1], adjustment [2], and maintenance of high-pressure pumps (HPP) in special-purpose vehicle engines, as presented in [3], [4], and [5]. Reliability and maintenance of high-pressure pumps (hereafter HPP) are crucial for the efficiency and longevity of engines in vehicles. Previous studies have explored various aspects of these pumps' reliability, including performance analysis and optimization methods. Papers [1] and [2] investigated reliability models for high pressures in the adjustment and testing of HPPs, while papers [3], [4], and [5] focused on the analysis and testing of different probability distributions to determine accurate reliability parameters and optimize maintenance systems. This paper continues the investigation into optimizing the maintenance of HPPs in special-purpose vehicles through the analysis of failure distributions and maintenance optimization criteria.

2. EXPERIMENT AND DISCUSSION

2.1. Reliability indicators estimation

Data on the time-to-failure of high-pressure pumps in engines, expressed in hours (h) and arranged in ascending order, are presented in Table 1.

Table 1: Time-to-Failure data for high-pressure pumps

Failure ordinal number	Time to failure [h]	Failure ordinal number	Time to failure [h]	Failure ordinal number	Time to failure [h]	Failure ordinal number	Time to failure [h]
1	51	24	275	47	386	70	548
2*	77	25	276	48	389	71	548
3	98	26	276	49	394	72	556
4	105	27*	277	50*	396	73	562
5	111	28*	278	51	399	74	579
6	147	29	284	52	400	75	590
7	152	30	284	53	403	76*	593
8	187	31	300	54	408	77*	604
9	197	32	301	55	413	78	618
10	199	33	316	56	415	79	619
11	201	34	318	57	428	80	628
12	201	35*	329	58	435	81	634
13	203	36	330	59	475	82	639
14*	204	37	333	60	475	83	662
15	206	38	338	61	475	84	699
16	233	39	339	62*	475	85	702
17*	238	40*	342	63*	476	86	735
18*	240	41	345	64*	478	87	754
19	240	42	348	65*	481	88	762
20	243	43	355	66*	488	89	763
21	246	44	362	67	512	90	847
22	251	45	368	68	532	91	905
23	272	46*	379	69	544	92	948

In Table 1, there are two types of results: one consists of the operating times of the high-pressure pump (HPP) motor until the occurrence of a failure condition, and the other consists of the operating times of the motor until it is removed from testing due to a failure of the motor that is not caused by the HPP. Failures of the motor caused by high-pressure pump failures (hereinafter referred to as HPP) are indicated by an asterisk next to the serial number of the failure (non-shaded fields) in Table 1.

Based on the data from Table 1, the reliability indicators (cumulative distribution function of probability - failure function, reliability function, failure intensity function, and failure density function) are estimated using methods for small samples and excluding results from motor failures not caused by HPP [6].

The estimation of reliability indicators and auxiliary quantities—incremental serial numbers and actual ranks of results—is conducted using the following formulas [6]:

- Ordinal number increment:

$$P = \frac{(Total\ number\ of\ results + 1) - Previous\ serial\ number}{1 + \ Number\ of\ results\ remaining\ after\ exclusion} \tag{1}$$

- Actual rank (ordinal number) of results: SR - calculated as the cumulative ordinal number increment P
- Median rank:

$$MR = \frac{(SR - 0.3)}{(n + 0.4)} \tag{2}$$

where n is the total number of results (n = 92, Table 1),

– Cumulative probability distribution function (reliability function):

$$F = MR \tag{3}$$

– Reliability function:

$$R = 1\text{-}F \tag{4}$$

– Failure intensity function:

$$h = \frac{1}{(n - SR + 0.7)\cdot \Delta t} \tag{5}$$

where $\Delta t = (t_j+1\text{-}t_j)$ - is the operating time between failures, and t_j – is the time to failure HPP,

– Failure frequency (density) function:

$$f = \frac{1}{(n + 0.4)\cdot \Delta t} \tag{6}$$

Estimated reliability indicators are given in Table 2.

Table 2: Estimated reliability indicators for the high-pressure pump

Failure ordinal number	Failure ordinal number of HPP	t[h]	P	SR	MR [%]	Δt[h]	F	R	h [h^{-1}]	f [h^{-1}]
1	2	77	1.0109	1.0109	0,7693	127	0.0077	0.9923	8.59E-05	8.52E-05
2	14	204	1.1499	2.1607	2.0138	34	0.0201	0.9799	3.25E-04	3.18E-04
3	17	238	1.1797	3.3405	3.2905	2	0.0329	0.9671	5.60E-03	5.41E-03
4	18	240	1.1797	4.5202	4.5673	37	0.0457	0.9543	3.06E-04	2.93E-04
5	27	277	1.3206	5.8408	5.9965	1	0.0600	0.9400	1.15E-02	1.08E-02
6	28	278	1.3206	7.1614	7.4257	51	0.0743	0.9257	2.29E-04	2.12E-04
7	35	329	1.4549	8.6163	9.0003	13	0.0900	0.9100	9.15E-04	8.33E-04
8	40	342	1.5627	10.1789	10.6915	37	0.1069	0.8931	3.28E-04	2.93E-04
9	46	379	1.7254	11.9044	12.5588	17	0.1256	0.8744	7.28E-04	6.37E-04
10	50	396	1.8431	13.7475	14.5535	79	0.1455	0.8545	1.60E-04	1.37E-04
11	62	475	2.4766	16.2241	17.2339	1	0.1723	0.8277	1.31E-02	1.08E-02
12	63	476	2.4766	18.7007	19.9142	2	0.1991	0.8009	6.76E-03	5.41E-03
13	64	478	2.4766	21.1774	22.5946	3	0.2259	0.7741	4.66E-03	3.61E-03
14	65	481	2.4766	23.6540	25.2749	7	0.2527	0.7473	2.07E-03	1.55E-03
15	66	488	2.4766	26.1307	27.9553	105	0.2796	0.7204	1.43E-04	1.03E-04
16	76	593	3.7150	29.8456	31.9758	11	0.3198	0.6802	1.45E-03	9.84E-04
17	77	604	3.7150	33.5606	35.9963	0	0.3600	0.6400	0	0

From Table 2, it can be seen that:

- The total number of data points (sample size) is: n=17,
- Minimum time to compressor failure: t_{min}=77 h
- Maximum time to compressor failure: t_{max}=604 h.

The statistical measures calculated from the data in Table 2 are:

- Mean time to failure: t_{sr}=373.8235 h
- Standard deviation of time to failure: SD=144.1506 h
- Median time to failure: median=379 h
- Range of time to failure: range= 527 h

Figures 1 to 4 show graphical representations of the estimated reliability indicators (unreliability function, reliability function, failure intensity function, and estimated values of failure state density).

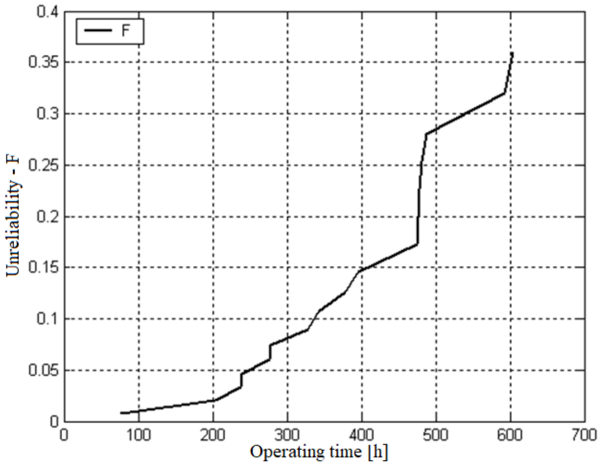

Figure 1: Graphical representation of the estimated values of the unreliability function

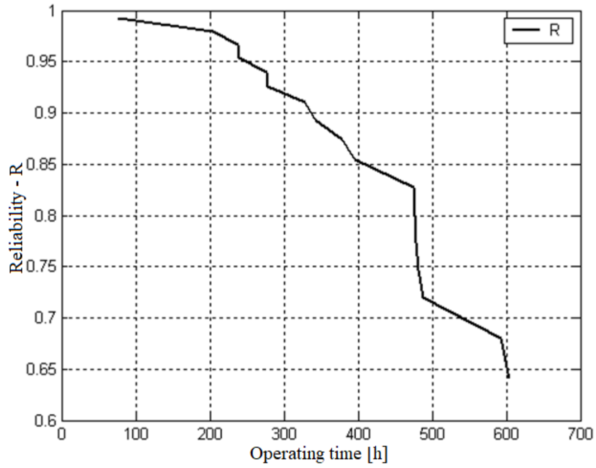

Figure 2: Graphical representation of the estimated values of the reliability function

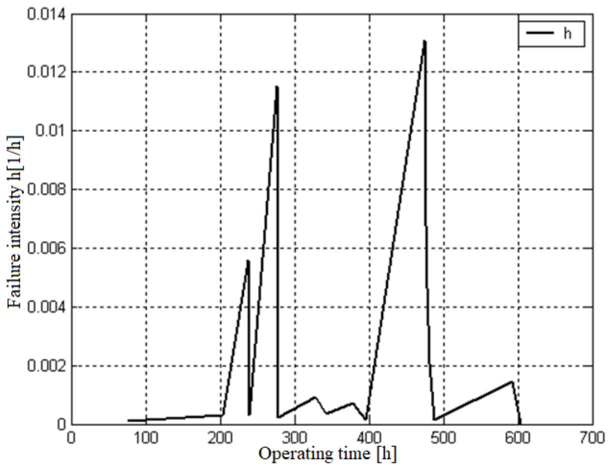

Figure 3: Graphical representation of the estimated values of the failure intensity function

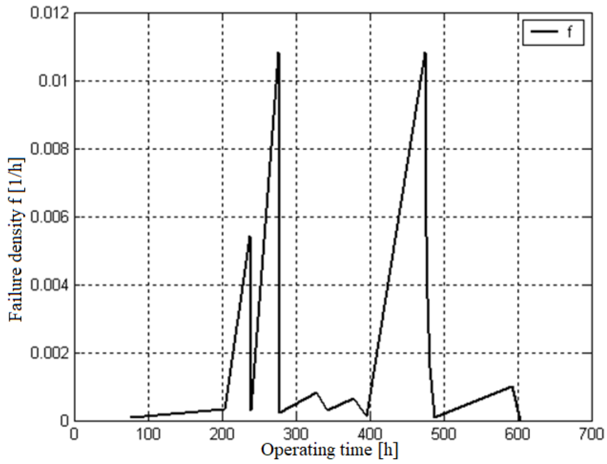

Figure 4: Graphical representation of the estimated values of the failure state density

2.2. Determination of Distribution Model and Parameters with Goodness-of-Fit Assessment

To determine a theoretical distribution model that could be used to approximate the empirical distribution, the empirical distribution is approximated using theoretical Weibull, exponential, and normal distributions. The goodness-of-fit between the empirical and theoretical distributions is assessed using the Kolmogorov-Smirnov test. Statistical data processing was performed using the Statistics Toolbox for MATLAB.

2.2.1. Approximation of the empirical distribution with a theoretical distribution

The hypothesis test for approximating the empirical distribution with one of the theoretical distributions was performed using graphical analysis methods [6]. The main idea is to plot the values obtained from system exploitation into the corresponding coordinate system based on the stated hypothesis. The axes of the coordinate system are adjusted so that the distribution appears as a straight line. Then, unlike the classical graphical method where the straight line is fitted based on a free estimate, the parameters of the regression line Y=aX+b are analytically determined using the method of least squares. The coefficients of the regression line are determined according to the equations:

$$a=\frac{\sum(X_i \cdot Y_i)-\frac{\sum X_i \cdot \sum Y_i}{n}}{\sum(X_i)^2-\frac{(\sum X_i)^2}{n}} \tag{7}$$

where:

n – number of result pairs X_i, Y_i, i=1,n,

$$b=\frac{\sum Y_i}{n}-a\frac{\sum X_i}{n} \tag{8}$$

This method also allows for the determination of the numerical value of the coefficient of determination (KD), which indicates the goodness of fit between the experimental results and the line defined by parameters aaa and bbb. The numerical value of the coefficient of determination is obtained using the following equation:

$$KD = \frac{\left(\sum(X_i \cdot Y_i) - \frac{\sum X_i \cdot \sum Y_i}{n}\right)^2}{\left(\sum(X_i^2) - \frac{(\sum X_i)^2}{n}\right) \cdot \left(\sum Y_i^2 - \frac{(\sum Y_i)^2}{n}\right)} \quad (9)$$

Once the line is defined through the given points, the numerical values of the distribution law parameters are determined. To assist with each attempt to determine the reliability distribution law in practice, a table is filled out, as shown in Table 3.

Table 3: Determination of distribution law parameters for reliability analysis

Ordinal No.i	Time to failure t [h]	Reliability – R (estimated)	Y	Y^2	X	X^2	X*Y
SUM:							

Based on the data from the completed Table 3 and equations (7), (8), and (9), the coefficients of the regression line a and b, as well as the coefficient of determination KD, are determined. Using the coefficients a and b, the unknown parameters of the assumed reliability distribution are determined, and then the values of the assumed theoretical distribution are calculated.

a) Approximation of the Empirical Distribution with the Weibull Distribution

In the case of the graphical-analytical method for the Weibull distribution, the X-axis is plotted with numerous values for www defined by the expression:

$$w = \ln\left(\ln\frac{1}{R(t)}\right) \quad (10)$$

where:

R(t) - is the estimated value of the reliability function, and

the Y -axis is plotted with values of ln(t).

Based on Table 2, Table 4 has been filled out in the same format as Table 3, for the Weibull distribution.

Table 4: Values for the Weibull Distribution

Serial No. of Compressor Failures - j	Time to Failure t [h]	Reliability R	$Y_w=\ln(t)$	Y_w^2	$X_w=\ln\left(\ln\frac{1}{R(t)}\right)$	X_w^2	$X_w Y_w$
1	77	0.9923	4.3438	18.8686	-4.8635	23.6540	-21.1262
2	204	0.9799	5.3181	28.2824	-3.8950	15.1710	-20.7141
3	238	0.9671	5.4723	29.9457	-3.3974	11.5426	-18.5917
4	240	0.9543	5.4806	30.0374	-3.0630	9.3817	-16.7870
5	277	0.9400	5.6240	31.6296	-2.7832	7.7464	-15.6529
6	278	0.9257	5.6276	31.6701	-2.5619	6.5633	-14.4173
7	329	0.9100	5.7961	33.5943	-2.3611	5.5749	-13.6852
8	342	0.8931	5.8348	34.0450	-2.1797	4.7512	-12.7182
9	379	0.8744	5.9375	35.2543	-2.0084	4.0336	-11.9249
10	396	0.8545	5.9814	35.7773	-1.8497	3.4215	-11.0640
11	475	0.8277	6.1633	37.9864	-1.6652	2.7729	-10.2632
12	476	0.8009	6.1654	38.0124	-1.5048	2.2643	-9.2774

Table 4: Values for the Weibull distribution (cont.)

Serial No. of Compressor Failures - j	Time to Failure t [h]	Reliability R	Y_w=ln(t)	Y_w^2	$X_w=\ln\left(\ln\frac{1}{R(t)}\right)$	X_w^2	$X_w Y_w$
13	478	0.7741	6.1696	38.0641	-1.3621	1.8554	-8.4038
14	481	0.7473	6.1759	38.1413	-1.2332	1.5208	-7.6162
15	488	0.7204	6.1903	38.3200	-1.1151	1.2434	-6.9028
16	593	0.6802	6.3852	40.7707	-0.9537	0.9096	-6.0897
17	604	0.6400	6.4036	41.0058	-0.8069	0.6511	-5.1672
		SUM:	99.0696	581.4056	-37.6041	103.0577	-210.4019

The coefficients a_w, b_w, and KD_w have been calculated using the data from Table 4 and equations (7), (8), and (9):

a_w =-0.1186,

b_w = 5.5653,

KD_w =0.9454.

The unknown parameters β and η of the Weibull distribution were determined using the following values [6]:

- $\beta=\frac{1}{a_w}=-8.4322$ – shape parameter,
- $\eta=e^{b_w}=261.2022$ – scale parameter.

To test the hypothesis about the approximation of the empirical distribution with the Weibull distribution, Table 5 was created. The hypothesis testing was performed using the Kolmogorov-Smirnov test.

Table 5: Values for testing the Weibull distribution

j	t	F	F_{tw}	ΔF
1	77	0.0077	0.0003	0.0074
2	204	0.0201	0.0074	0.0127
3	238	0.0329	0.0121	0.0208
4	240	0.0457	0.0124	0.0333
5	277	0.0600	0.0195	0.0405
6	278	0.0743	0.0197	0.0546
7	329	0.0900	0.0333	0.0567
8	342	0.1069	0.0376	0.0693
9	379	0.1256	0.0517	0.0739
10	396	0.1455	0.0592	0.0864
11	475	0.1723	0.1028	0.0695
12	476	0.1991	0.1035	0.0957
13	478	0.2259	0.1048	0.1212
14	481	0.2527	0.1068	0.1460
15	488	0.2796	0.1115	0.1681
16	593	0.3198	0.1967	0.1230
17	604	0.3600	0.2072	0.1527
			Max(ΔF)	**0.1681**

The labels used in Table 5 have the following meanings:

- j – serial number of failures,
- t – time to failure of the high-pressure pump [h],
- F – cumulative probability distribution function (unreliability function) – estimated values,
- F_{tw} – approximate Weibull (theoretical) cumulative probability distribution function,
- $\Delta F=\left|F-F_w\right|$ – absolute value of the difference between the empirical and theoretical cumulative distributions.

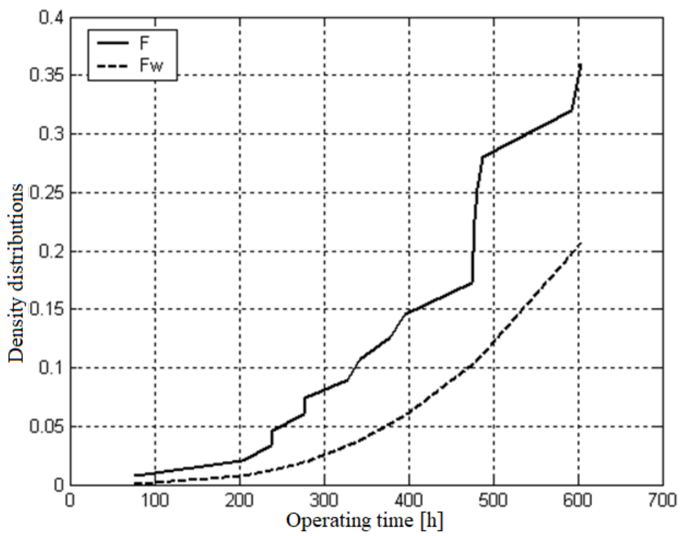

Figure 5: Display of Deviation of the Weibull Approximate Distribution from the Empirical Distribution

Kolmogorov-Smirnov Test:

For the adopted risk level α=0.2 and sample size N=n=17, the tabulated value for $d_{\alpha}=D_{threshold}=0.250$. According to Table 5, the maximum difference between the theoretical distribution (F_{tw}) and the estimated values (F) is: $Max(\Delta F)=D_n=0.1681$.

Since $D_n<D_{threshold}$, the theoretical approximate Weibull distribution satisfies the Kolmogorov-Smirnov test.

a) Approximation of the Empirical Distribution with the Exponential Distribution

For the exponential distribution, in applying the graphical-analytical method, the X-axis is plotted with numerous values defined by the expression $X_e=\ln(R(t))$, where R(t) is the estimated value of the reliability function, and the Y-axis is plotted with values $Y_e=t$. Based on Table 2, Table 6 is filled out in the format of Table 3 for the exponential distribution.

Table 6: Values for the Exponential Distribution

Serial No. of Compressor Failures j	Time to Failure t [h]	Reliability R	Ye=t	Y_e^2	Xe = ln(R(t))	X_e^2	Xe Ye
1	77	0.9923	77	5929	-0.0077	0.0001	-0.5947
2	204	0.9799	204	41616	-0.0203	0.0004	-4.1500
3	238	0.9671	238	56644	-0.0335	0.0011	-7.9632
4	240	0.9543	240	57600	-0.0467	0.0022	-11.2198
5	277	0.9400	277	76729	-0.0618	0.0038	-17.1292
6	278	0.9257	278	77284	-0.0772	0.0060	-21.4502
7	329	0.9100	329	108241	-0.0943	0.0089	-31.0293
8	342	0.8931	342	116964	-0.1131	0.0128	-38.6711
9	379	0.8744	379	143641	-0.1342	0.0180	-50.8634
10	396	0.8545	396	156816	-0.1573	0.0247	-62.2829
11	475	0.8277	475	225625	-0.1892	0.0358	-89.8469
12	476	0.8009	476	226576	-0.2221	0.0493	-105.7062
13	478	0.7741	478	228484	-0.2561	0.0656	-122.4221
14	481	0.7473	481	231361	-0.2914	0.0849	-140.1415
15	488	0.7204	488	238144	-0.3279	0.1075	-160.0069
16	593	0.6802	593	351649	-0.3853	0.1485	-228.4868
17	604	0.6400	604	364816	-0.4462	0.1991	-269.5226
		SUM:	6.355	2.708.119	-2.8643	0.7686	-1361.4869

The coefficients a_e, b_e and KD_e have been calculated using the data from Table 6 and equations (7), (8), and (9):

- a_e=−1.016.4213
- b_e=202.5713
- KD_e=0.8889

The unknown parameter λ (mean) of the exponential distribution was determined using the following equation [6]:

$$\lambda = |a_e| + b_e = 1218.9923 \quad (11)$$

To test the hypothesis about the approximation of the empirical distribution with the exponential distribution, Table 7 has been created. The hypothesis testing is performed using the Kolmogorov-Smirnov test.

Table 7: Values for testing the Exponential distribution

j	t	F	F_{te}	ΔF
1	77	0.0077	0.0419	0.0342
2	204	0.0201	0.1071	0.0870
3	238	0.0329	0.1238	0.0909
4	240	0.0457	0.1248	0.0791
5	277	0.0600	0.1426	0.0826
6	278	0.0743	0.1431	0.0688
7	329	0.0900	0.1670	0.0770
8	342	0.1069	0.1730	0.0661
9	379	0.1256	0.1898	0.0642
10	396	0.1455	0.1974	0.0519
11	475	0.1723	0.2319	0.0596
12	476	0.1991	0.2323	0.0332
13	478	0.2259	0.2332	0.0072
14	481	0.2527	0.2344	0.0183
15	488	0.2796	0.2374	0.0421
16	593	0.3198	0.2806	0.0391
17	604	0.3600	0.2850	0.0750
			Max(ΔF)	**0.0909**

The labels used in Table 7 have the following meanings:

- j – serial number of failures,
- t – time to failure of the compressor [h],
- F – cumulative probability distribution function (unreliability function) – estimated values,
- F_{te} – approximate exponential (theoretical) cumulative probability distribution function,
- $\Delta F = |F - F_{te}|$ - absolute value of the difference between the empirical and theoretical cumulative distributions.

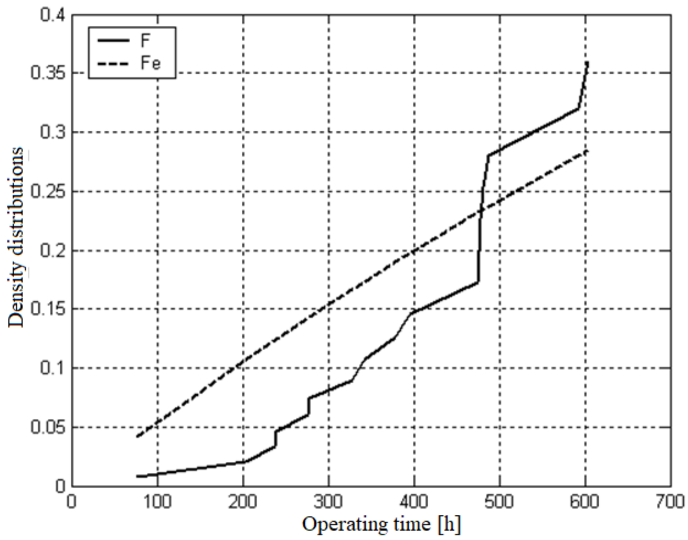

Figure 6: Deviation of the exponential approximate distribution from the empirical distribution

Kolmogorov-Smirnov Test:

For the adopted risk level α=0.2 and sample size N=n=17, the tabulated value for $d_\alpha=D_{threshold}=0.250$. According to Table 7, the maximum difference between the theoretical distribution (F_{te}) and the estimated values (F) is: Max(ΔF)=Dn=0.0909.

Since $D_n<D_{threshold}$, the theoretical approximate exponential distribution satisfies the Kolmogorov-Smirnov test.

a) Approximation of the Empirical Distribution with the Normal Distribution

When applying the graphical-analytical method to the normal distribution, the X-axis is plotted with numerous values for z defined by the expression:

$$z = p - \frac{c_0 + c_1 \cdot p + c_2 \cdot p^2}{1 + d_1 \cdot p + d_2 \cdot p^2 + d_3 \cdot p^3} \tag{12}$$

$$p = \sqrt{\ln\left(\frac{1}{R(t)}\right)} \tag{13}$$

Where:

R(t) is estimated relability funtion, and constants have the following values:

c_0 =2.515517, c_1 =0.802853, c_2 =0.00328, d_1 =1.432788, d_2 =0.189269 and d_3 =0.001308.

The Y-axis is plotted with values for the time to failure t. Based on Table 2, Table 8 has been filled out in the format of Table 3 for the normal distribution.

Table 8: Values for the Normal Distribution

Serial No. of Compressor Failures j	Time to Failure t [h]	Reliability R	Y_n=t	Y_n^2	X_n = z	X_n^2	X_n Y_n
1	77	0.9923	77	5.929	-2.2061	4.8668	-169.8675
2	204	0.9799	204	41.616	-2.0343	4.1385	-415.0045
3	238	0.9671	238	56.644	-1.9163	3.6723	-456.0834
4	240	0.9543	240	57.600	-1.8234	3.3249	-437.6244
5	277	0.9400	277	76.729	-1.7365	3.0155	-481.0198
6	278	0.9257	278	77.284	-1.6614	2.7602	-461.8691
7	329	0.9100	329	108.241	-1.5881	2.5221	-522.4894
8	342	0.8931	342	116.964	-1.5175	2.3028	-518.9833
9	379	0.8744	379	143.641	-1.4468	2.0933	-548.3415
10	396	0.8545	396	156.816	-1.3778	1.8982	-545.5950
11	475	0.8277	475	225.625	-1.2930	1.6718	-614.1597
12	476	0.8009	476	226.576	-1.2151	1.4766	-578.4112
13	478	0.7741	478	228.484	-1.1427	1.3057	-546.1998
14	481	0.7473	481	231.361	-1.0744	1.1543	-516.7861
15	488	0.7204	488	238.144	-1.0095	1.0190	-492.6186
16	593	0.6802	593	351.649	-0.9169	0.8408	-543.7444
17	604	0.6400	604	364.816	-0.8288	0.6869	-500.6033
		SUM:	6.355	2.708.119	-24.7887	38.7498	-8349.4012

The coefficients a_n, b_n, and KD_n have been calculated using the data from Table 8 and equations (7), (8), and (9):

- a_n =352.2313,
- b_n=887.4325,
- KD_n =0.9717

The unknown parameters μ and σ of the normal distribution were determined using the following equations [6]:

- μ=b_n=887.4325 – mean
- σ=a_n=352.2313 – standard deviation

To test the hypothesis about the approximation of the empirical distribution with the normal distribution, Table 9 has been created. The hypothesis testing is performed using the Kolmogorov-Smirnov test.

Table 9: Values for testing the Normal distribution

j	t	F	F_{tn}	ΔF
1	77	0.0077	0.0065	0.0012
2	204	0.0201	0.0144	0.0057
3	238	0.0329	0.0176	0.0153
4	240	0.0457	0.0178	0.0278
5	277	0.0600	0.0220	0.0379
6	278	0.0743	0.0222	0.0521
7	329	0.0900	0.0293	0.0607
8	342	0.1069	0.0314	0.0755
9	379	0.1256	0.0381	0.0875
10	396	0.1455	0.0416	0.1040
11	475	0.1723	0.0610	0.1113
12	476	0.1991	0.0613	0.1379
13	478	0.2259	0.0619	0.1641
14	481	0.2527	0.0627	0.1900
15	488	0.2796	0.0648	0.2148
16	593	0.3198	0.1024	0.2174
17	604	0.3600	0.1071	0.2529
			Max(ΔF)	**0.2529**

The labels used in Table 9 have the following meanings:

- j – serial number of failures,
- t – time to failure of the high-pressure pump [h],
- F – cumulative probability distribution function (unreliability function) – estimated values,
- F_{tn}– approximate normal (theoretical) cumulative probability distribution function,
- $\cdot F = \left|F - F_{tn}\right|$ – absolute value of the difference between the empirical and theoretical cumulative distributions.

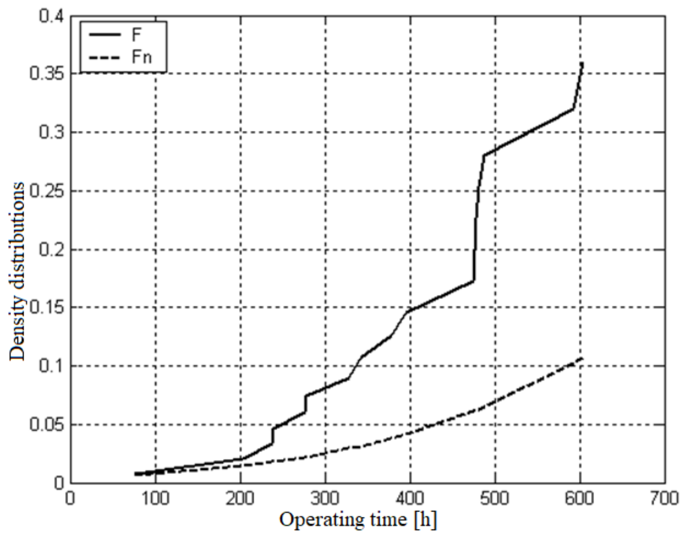

Figure 7: Graphical representation of the deviation of the normal approximate distribution from the empirical distribution

Kolmogorov-Smirnov Test:

For the adopted risk level α=0.2 and sample size N=n=17, the tabulated value for $d_{\alpha}=D_{threshold}=0.250$. According to Table 9, the maximum difference between the theoretical distribution (F_{tn}) and the estimated values (F) is: Max(ΔF)=D_n=0.2529.

Since $D_n>D_{threshold}$, the theoretical approximate normal distribution DOES NOT satisfy the Kolmogorov-Smirnov test.

From the previous analysis, we conclude that the Weibull and exponential distributions satisfy the test, while the normal distribution does not. Since the maximum ΔF is smallest in Table 7, we adopt the exponential distribution with a mean λ=1.218.9926 as the approximate reliability model for the high-pressure pump (HPP) engine. Therefore, the expression for the probability of correct operation (reliability function) of the HPP engine is:

$$R(t)=e^{-\frac{1}{\lambda}}=e^{-\frac{t}{1218.9926}} \quad (14)$$

When calculating the probability of correct operation using this expression, the variable ttt is expressed in hours [h] of operation.

2.3. Determination of maintenance periodicity of the HPP engine based on maximum readiness criterion

For special-purpose vehicles, the optimization criterion based on the maximum readiness model is the most appropriate. To apply the readiness-based maintenance model, knowledge of the reliability distribution law, as well as the operating and failure times, is required. The value of operational readiness can be determined using the expression [7]:

$$G(t)=\frac{t_r+t_{cr}}{t_r+t_{cr}+t_p+\frac{F(t)}{R(t)}\cdot t_k} \quad (15)$$

where:

- t_r – operating time,
- t_{cr} – waiting time in correct operation,
- t_p – time for preventive maintenance,
- t_k– time for corrective maintenance,
- R(t) – reliability function,
- F(t)– unreliability function.

By varying the periodicity of time between preventive maintenance, a functional dependence of readiness on maintenance periodicity is obtained, which allows determining the maintenance periodicity that provides maximum readiness. The results of readiness determination for different maintenance periodicities are given in Table 10.

Table 10: Results of readiness determination for different maintenance periodicities

Maintenance Periodicity [h]	250	300	350	400	450	500	550	600	650	700
Operating Time t_r [h]	250	300	350	400	450	500	550	600	650	700
Preventive Maintenance Time t_p [h]	150	150	150	150	150	150	150	150	150	150
Unreliability Function F(t)	0.1854	0.2182	0.2496	0.2797	0.3087	0.3365	0.3631	0.3887	0.4133	0.4369
Reliability Function R(t)	0.8146	0.7818	0.7504	0.7203	0.6913	0.6635	0.6369	0.6113	0.5867	0.5631
No. of Corrective Maintenances Between Two Preventive n_k	0.2276	0.2790	0.3326	0.3884	0.4465	0.5071	0.5702	0.6359	0.7044	0.7758
Corrective Maintenance Time t_k [h]	170.72	209.28	249.44	291.29	334.89	380.31	427.64	476.95	528.32	581.84
Waiting Time in Correct Operation t_{cr} [h]	1000	1200	1400	1600	1800	2000	2200	2400	2600	2800
Readiness G(t)	0.8687	0.8780	0.8825	0.8837	0.8825	0.8794	0.8747	0.8687	0.8616	0.8534

From Table 10, we see that the maximum readiness is within the operating time t_r interval of 350 to 450 hours. We discretize the maintenance periodicity interval (i.e., operating time t_r) from 250 to 700 with a step of 1 and calculate the maximum readiness value and the corresponding maintenance periodicity for maximum readiness. Figure 8 shows the Matlab code used to discretize the maintenance periodicity interval from 250 to 700 hours and calculate the maximum readiness value. The optimal preventive maintenance periodicity for achieving maximum readiness of 0.8837 is determined to be 398 hours, as illustrated in the code outputs and corresponding graphical representation.

```
Matlab
tr = 250:700;
tp = 150;
R = exp(-(tr/1218.9926));
F = 1 - R;
nk = F ./ R;
tk = 5 * tp * nk;
tcr = 1000:4:2800;
G = (tr + tcr) ./ (tr + tcr + tp + tk .* F ./ R);
i = find(G == max(G));
i = 149;
max(G) = 0.8837;
tr(149) = 398;
```

Figure 8: Matlab code used for discretize the maintenance periodicity interval

It is concluded that the maximum readiness max(G)=0.8837 is achieved for the preventive maintenance periodicity of tr (149) = 398 operating hours. Figure 9 shows the dependence of readiness on the periodicity of preventive maintenance for the HPP engine. The figure illustrates how readiness varies with different maintenance intervals, with the optimal preventive maintenance periodicity of 398 hours achieving the maximum readiness of 0.8837.

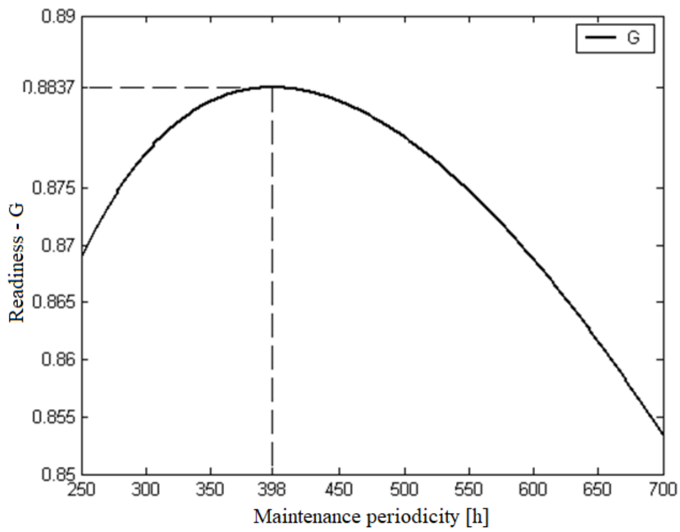

Figure 9: Dependence of readiness on the periodicity of preventive maintenance for the HPP engine

3. CONCLUSION

The results presented in this work provide a modest contribution to determining the reliability model and establishing the optimal periodicity for preventive maintenance of high-pressure pumps used in special-purpose engines. Based on the adopted reliability model, the time for preventive maintenance of high-pressure pumps can be determined. The results indicate that maximum readiness is achieved with a preventive maintenance interval of approximately 398 hours of engine operation for special-purpose vehicles. The cost minimization criterion has not been specifically optimized due to the nature of the exponential distribution, as there is no optimal time that ensures minimal maintenance costs for failures following an exponential distribution law. This work lays the groundwork for improving the maintenance of high-pressure pumps through the application of statistical methods and reliability models. The presented methodology can also be applied to other technical equipment.

REFERENCES

[1] Ž. M. Bulatović, D. M. Knežević, S. Lj. Biočanin, and M. S. Timotijević, "Testing high-pressure pumps on the BOSCH test bench", Engineering Today, Vol.3(2), pp. 25-33, https://doi.org/10.5937/engtoday2400005B, (2024)

[2] Ž. M. Bulatović, D. M. Knežević, S. Lj. Biočanin, and M. S. Timotijević, "Adjustment and regulation of high-pressure pumps on the BOSCH test bench", Engineering Today, Online First, https://doi.org/10.5937/engtoday2400008B, (2024)

[3] S. Lj. Biočanin and M. S. Timotijević, "A reliability analysis of the horizontal milling machine: GVK – 1P", IMK-14 – Research & Development in Heavy Machinery, Vol. 26(1), pp. 1-6, https://doi.org/10.5937/IMK2001001B, (2020)

[4] S. Lj. Biočanin and M. S. Timotijević, „Odredjivanje modela pouzdanosti vozila posebne namene", Zbornik radova šestog naučno-stručnog skupa "Politehnika", Belgrade (Serbia), pp. 474-479, (2021)

[5] B. Krstić, R. Nikolić, S. Biočanin, I. Krstić, and V. Krstić, "Determination of the driving engine reliability for the special purposes motor vehicle", Proceedings of the 10th International Scientific Conference on Sustainable, Modern, and Safe Transport TRANSCOM 2013, Žilina (Slovakia), (2013)

[6] J. Knežević, "Upravljanje procesima održavanja i obnavljanja tehničkih sistema na osnovu teorije pouzdanosti", OMO, Belgrade (Serbia), (1988)

[7] B. Krstić, "Eksploatacija motornih vozila i motora", Mašinski fakultet, Kragujevac (Serbia), (1997)