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Contribution to the development of a method for dimensioning the torsion stabilizer of a bus in the phase of the conceptual design

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ABSTRACT

During the conceptual design phase of a bus, many parameters are unknown and are adopted based on analogous design solutions or relevant recommendations. This paper develops a method for defining the diameter of the torsion stabilizer with the aim of minimizing its forced torsional vibrations and torsional loads, using a "stochastic parameter optimization" method. During the post-optimal analysis, the diameter was calculated based on the allowable rolling angle of the bus body, and the construction safety factor of the stabilizer was calculated. The method is illustrated with the mass and dimensional parameters of the intercity bus SANOS S 13, with a total mass of 13520 kg. The analysis determined that the developed method yields acceptable results, as comparisons were made with the dimensions of the torsion stabilizer of the produced bus.

KEYWORDS

Bus, Torsion stabilizer, Torsional vibrations, Optimization, Frequency analysis.

1. INTRODUCTION

This paper will discuss automatic design, more precisely, the definition of the dimensions of the bus torsion stabilizer from the perspective of minimizing forced damped torsional vibrations and torsional loads [1]. The design task defined that an intercity bus with a total mass of 13520 kg should be designed for the market. The torsion stabilizer serves to limit the rolling of the bus body when traveling on uneven roads or on curved paths. In practice, there are various conceptual designs with one or more torsion stabilizers [2]. For illustration, Figure 1. shows one possible design solution for the torsion stabilizer [1], where a_l and a_r denote the arms, b_l and b_r denote the supports, and TS denotes the torsion bar.

For instance, the SANOS S13 bus is equipped with only one stabilizer, located in the rear suspension system [2]. Therefore, the parameters of this bus will be used as analogs during dynamic simulation. It is important to note that the torsion stabilizer is subjected to torsion due to the rolling of supported and unsupported masses. To automatically define the dimensions of the torsion stabilizer at this stage of the project, both a physical model (Figure 2.) and a corresponding mathematical model will be used. As is well known, optimal design is based on minimization methods for the objective function, and in our case, the stochastic parameter optimization procedure based on the Hooke-Jeeves method was employed [3-5].

Analyses have shown that the material of the torsion stabilizer is most fatigued due to prolonged vibrational loads [6]. Practice shows that buses are primarily used under driving conditions when torsional vibrations occur due to the rolling of suspended and unsuspended masses. Bearing this in mind, the method is based on minimizing torsional vibrations and torsional loads under those conditions. In conditions of rigorous operation, during cornering, lateral accelerations of up to 30% of gravitational acceleration are allowed [1,2], so it is necessary to verify the dimensions of the torsion stabilizer under such exploitation conditions and calculate its construction safety factor.

2. METHOD

For the study of torsional vibrations of the stabilizer, it was deemed appropriate to idealize it and treat it as a homogeneous bar of adopted length with an unknown radius [7,8] (Figure 2). The torsion stabilizer undergoes forced torsional vibrations under the influence of disturbing torques acting at its ends. Bearing this in mind, further discussion will focus on modeling the torsional vibrations of the torsion stabilizer. It should be emphasized that the adopted concept of the stabilizer consists of two rigid arms and an elastic torsion bar (Figure 1). Therefore, this paper will develop a method for dimensioning the torsion bar of the stabilizer.

2.1. Model of the torsion bar of the stabilizer

When defining the model that describes the forced torsional vibrations of the elastic torsion stabilizer, the following assumptions were made:

- The stabilizer consists of three parts: the torsion bar and two arms at the ends (Figure 1),
- The arms have high rigidity, so they can be assumed to be non-deformable in relation to the torsion bar,
- The torsion bar is homogeneous and of constant diameter, and
- The existence of other vibrational excitations originating from the vehicle itself is neglected.

Considering the primary function of the stabilizer to reduce the rolling of the suspended mass, it should be noted that this occurs due to the torsion that develops within it, under the influence of torques acting on the ends of the stabilizer. Its main task is to prevent significant tilting of the vehicle around its longitudinal axis (rolling) – it is commonly defined that the maximum rolling angle should be between 4° - 6° [1,2].

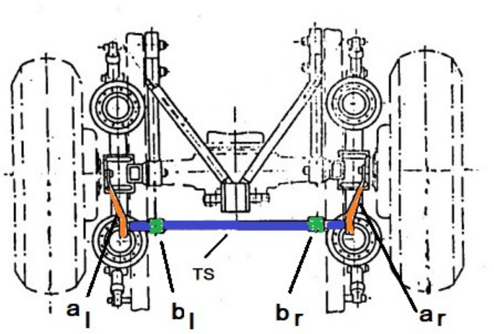

Figure 1: One constructive design of the stabilizer in a bus

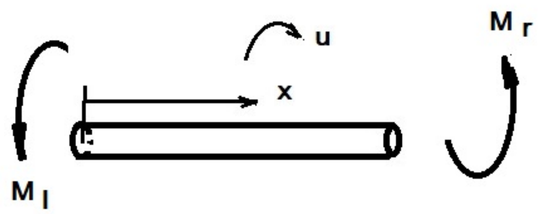

Figure 2: Model of the torsion bar of the stabilizer

It was deemed appropriate to calculate the torques based on the inertial forces that occur at the ends of the stabilizer arms. As recorded in [9], vertical accelerations at the connection points of the stabilizer arms with the rear axle were measured, and the magnitudes of the inertial forces were calculated by multiplying the recorded accelerations and unsuspended mass of the rear axle of the bus. Based on the calculated inertial forces, the torsional torques at the ends of the torsion bar were obtained by multiplying them by the length of the stabilizer arms. For illustration, these moments are shown in Figure 3, which reveals that the torsional torques are of a stochastic nature and that there is a phase shift between them.

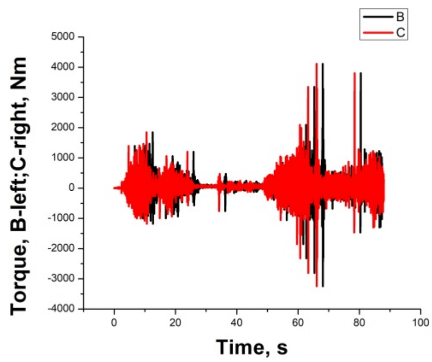

Figure 3: Torsional moments at the ends of the torsion bar of the stabilizer

Since the partial differential equation describing the forced torsional vibrations of the elastic torsion bar is detailed in [7,8], it will not be repeated here, but its final form will be presented. The forced torsional vibrations of the torsion bar are described by the partial differential equation:

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial x^2} + f(x,t) \tag{1}$$

where:

- u(x,t) - torsional vibrations of the torsion bar,
- x - coordinate along the length of the torsion bar,
- f(x,t) - difference of the forced torsional torques at the ends of the bar shown in Figure 3,
- t - time,
- G - shear modulus, and
- ρ - density of the stabilizer material.

As is known [7,8], in order to determine the general integral of the partial differential equation (1), it is necessary to know the boundary and initial conditions. The torsional torque due to vibrations of the torsion bar can be expressed [6]:

$$M = GI_0 \frac{\partial u(x,t)}{\partial x} \tag{2}$$

where:

- I_0 - polar moment of inertia of the cross-section of the torsion bar, given by the expression [6]:

$$I_0 = \frac{\pi r^4}{2}$$

where:

- r - radius of the torsion bar.

Defining boundary conditions in the analysis of elastic body vibrations, in general, represents an idealization of the real state. In this specific case, it is assumed that both ends are quasi-free and subjected to forced torsional moments. Additionally, it is assumed that the torsional vibrations and their velocities are zero at the initial torque, i.e:

$$\begin{gathered} GI_0 \frac{\partial u(L,t)}{\partial x} = M_l \\ GI_0 \frac{\partial u(L,t)}{\partial x} = M_r \\ u(x,0) = 0 \\ u'(x,0) = 0 \end{gathered} \tag{3}$$

where:

- M_l, M_r – forced (disturbing) torqes the left and right ends of the torsion bar, respectively, and
- L – its length.

The partial differential equation (1), along with the boundary and initial conditions (3), cannot be solved in a closed form. Therefore, it was solved numerically [10], using the finite difference method. Since this procedure is known from [10], it will not be discussed further here. Using the mentioned method, the author developed a program in Pascal and numerically solved the partial differential equation (1) with the excitation function f(x,t) and the boundary and initial conditions (3). It should be noted that in the case of numerically solving partial differential equations, it is, sometimes, necessary to introduce additional boundary and initial conditions [11]. Dynamic simulation and post-optimal analysis were carried out with the data provided in Table 1.

Table 1: Data used during the research

Quantity	Value	Quantity	Value
Shear modulus, G, N/mm^2	8x10^4	Half-track of the front springs, b_1, mm	450
Density of the material, ρ, kg/mm^3	8x10^{-6}	Half-track of the rear springs, b_2, mm	465
Number of points along the length, n_x	2048	Stiffness of the front springs, c_1, N/mm	1.02x10^8

Quantity	Value	Quantity	Value
Number of points in time, n_t	2048	Stiffness of the rear springs, c_2, N/mm	2.62x10^8
Step length, hx, mm	0.5	Length of the torsion bar, L, mm	640
Time step, h_t, s	0.02	Length of the stabilizer arms, L_p, mm	1024
Total mass of the bus, mm, kg	13520	Allowed tangential stress, τ_a, MPa	300
Front unsuspended mass, m_1, kg	726	Distance from the center of gravity of the suspended mass to the rolling axis, h_v, mm	550
Rear unsuspended mass, m_2, kg	1334		

The values used for the number of points and discretization steps during the dynamic simulation provided reliability for the parameters in the x-direction: 0.000976 – 1, 1/mm, and time: 0.0244 – 25, Hz [12].

3. AUTOMATIC SELECTION OF THE RADIUS OF THE TORSION BAR

Various optimization procedures are used in practice, and here the "stochastic parameter optimization" method will be applied, as detailed in [3-5]. Constructive constraints for the optimizing parameters were introduced through the application of "external penalty functions" [3-5].

In this specific case, the selection of the radius of the torsion bar (with the length limited by the rear spring track, at L=1024, mm) utilized the Hooke-Jeeves method. For illustration of the optimization process, Figure 4 shows a block diagram (the software was developed by the author in Pascal). Since the description of the optimization method is detailed in [3,4], it will not be discussed further in this paper (the Figure is provided for better understanding by the readers). It was deemed appropriate to calculate the radius of the torsion bar based on the minimization of its forced vibrations described by expression (1) and the minimization of torsional loads.

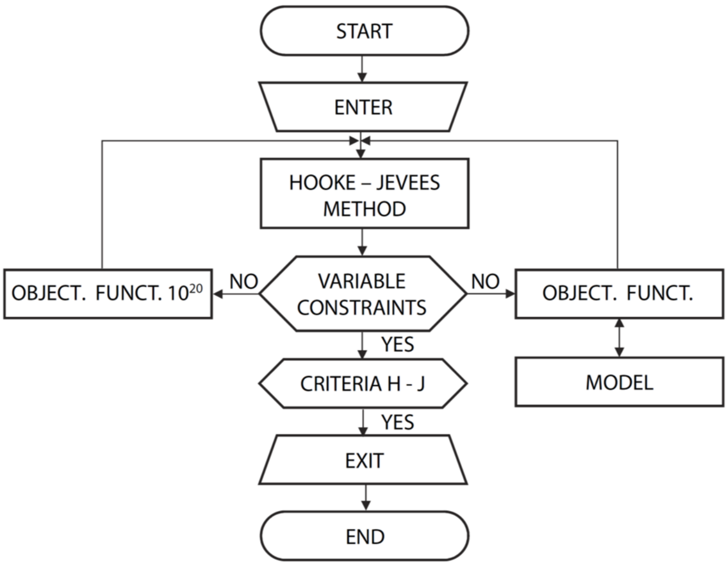

Figure 4. Block diagram of the optimization method used

With this in mind, the objective function of the form was used:

$$Z = u_{RMS} + \tau_{RMS} \quad (4)$$

where:

– u_{RMS} - values of the forced vibrations of the torsion bar obtained by solving the PDE (1),
– τ_{RMS} - torsional load of the torsion bar, calculated based on expression (2).

The RMS is calculated using the expression:

$$u_{RMS}{}^2 = \frac{1}{n_x n_t} \sum_{i=1}^{nx} \sum_{j=1}^{nt} u(i,j)^2$$
$$\tau_{RMS}{}^2 = \frac{1}{n_x n_t} \sum_{i=1}^{nx} \sum_{j=1}^{nt} \tau(i,j)^2 \quad (5)$$

where:

– u(i,j) - torsional vibrations of the torsion bar,

- $\tau(i,j)$ - torsional stress in the torsion bar,
- n_x - number of points along the x-axis, and
- n_t- number of points along the t-axis.

During the optimal dimension selection process for the torsion bar, boundary values for its radius were defined:

$$10 \le r \le 40$$

By introducing the optimizing parameter x[i] i=1, instead of r, with the corresponding adopted boundary values $x_u[i]$ and $x_l[i]$ (where u is the upper and l is the lower boundary value), the objective function depends on one optimizing parameter and has multiple local minima but only one global minimum [5].

Since there is no precise procedure for determining the global minimum, the optimization process was initiated with three initial values of the optimizing parameter [3,4], specifically:

$$\begin{aligned} x &= 0.5x_u[1]; \qquad & x &= 0.5x_u[2] \\ x &= 0.8x_u[1]; \qquad & x &= 0.8x_u[2] \\ x &= 1.2x_l[1]; \qquad & x &= 1.2x_l[2] \end{aligned}$$

The optimal radius should be adopted as the one for which the value of the objective function is the smallest. The optimization was performed on a Pentium 4 computer (Intel 2.4 GHz; 9 GB RAM), and the iterative process was automatically terminated when the difference between two neighboring values of the objective function was 10^{-10}. The optimization time for one combination was approximately 40 minutes, and the calculated parameters are presented in Table 2.

Table 2. The calculated parameters of torsion bar

Initial values	**Optimal parameter** r [mm]	**Objective function** Z	**Number of iterations** N
$0.5(x_l+x_u)$	1.000000000953459E+01	1.236283346694984E-002	460
$0.8x_u$	1.000000000953459E+01	1.236283346694560E-002	524
$1.2x_l$	1.000000000953459E+01	1.236283341984032E-002	182

4. DATA ANALYSIS

Analyzing the data from Table 2, it can be concluded that the optimal value of the objective function is only slightly dependent on the initial value of the optimizing parameter, while the optimal parameter value is identical for the specified accuracy of 10^{-10}. The only difference is the number of iterations during the calculation of the optimal value.

4.1. Post - optimal analysis

It was deemed appropriate to verify the calculated radius of the torsion bar during the bus's motion along a circular path. The literature provides procedures for an orientational selection of the radius of the stabilizer based on the condition that the lateral acceleration of the center of gravity should be a maximum of 30% of the acceleration due to gravity [1,2]. More precisely, the torsional stiffness of the stabilizer's torsion bar can be calculated using the expression [2]:

$$c_{st} = \frac{0.3 g_e (m - m_1 - m_2)}{\varphi_m} - 2(c_1 b_1^2 + c_2 b_2^2) \tag{6}$$

where:

- g_e– acceleration due to gravity,
- m=m1+m2 – the magnitude of the suspended mass calculated from the data in Table 1,
- φ_m – the maximum allowed value of the roll angle of the suspended mass, which ranges from 3° - 6°; a value of 6° is adopted,
- c_1, c_2– installed stiffness of the springs; typically, the stiffness of the free spring is increased by 20-30% (in some cases, 50%); here, an increase of 20% compared to the values given in Table 1 is assumed.

Taking the above explanations and the data from Table 1 into account, using expression (5), the torsional stiffness of the stabilizer's torsion bar was calculated to be 132440 Nm/rad, which corresponds to a stabilizer radius of 18015 mm.

It is also important to check the construction safety factor for the calculated radius of the stabilizer's torsion bar. It is defined as [6]:

$$\nu = \frac{\tau_a}{\tau} \tag{7}$$

where:

– τ_a – allowed torsional stress, and
– τ – actual torsional stress in the stabilizer.

The magnitude of the actual stress can be calculated using expression [6]:

$$\tau = \frac{16Ms}{\pi d^3}$$

where:

– M_s – actual torsional torque in the stabilizer.

The torsional stiffness is given by the expression [6]:

$$c_{st} = \frac{G\pi d^4}{32L} \tag{8}$$

For a diameter of d=36.03 mm and a length of 1024, mm, it amounts to 12882, Nm/rad. Based on the torsional stiffness, the torsional moment is calculated using the expression:

$$M_{st} = c_{st}\varphi_a \tag{9}$$

where:

– φ_m – adopted maximum roll angle of the suspended mass of 6°.

Using expressions (7, 8) and the data from Table 2, the construction safety factor of the torsion stabilizer was calculated to be 3.73. Since the safety factor for motor vehicles is typically chosen within the range of 2–3 [1], it is clear that it is fully satisfactory. Based on this, it can be stated that the developed procedure for selecting the (semi) diameter of the stabilizer's torsion bar during the conceptual design phase yields acceptable results.

It should be noted that based on the pre-established construction safety factor, the diameter of the stabilizer's torsion bar can be calculated using a reverse procedure, which is not done here; the largest of the calculated diameters should be adopted as optimal. It was deemed useful to analyze the vibrations of the stabilizer's torsion bar for the optimal radius. By solving equation (1), the data shown in Figure 5 were obtained.

Analyzing the data from Figure 5, it can be determined that the torsional vibrations change stochastically over time. The random nature of the vibrations can be explained by the stochastic character of the applied excitation function, which is consistent with [7,8]. It was deemed appropriate to perform a frequency analysis of the torsional vibrations of the bar using 2D Fourier transform [3] with the commercial software Origin [13]. The calculated values of magnitude and phase angles from the 2D Fourier transform are shown in Figures 6 and 7.

It should be noted that an analysis of torsional resonance of the torsion bar must be performed based on the adopted standard dimensions and compared with the harmonic frequencies obtained through 2D Fourier transformation. The goal is to ensure that there is no overlap between the resonance of the torsion bar and the frequencies of the waves, which will not be discussed further here.

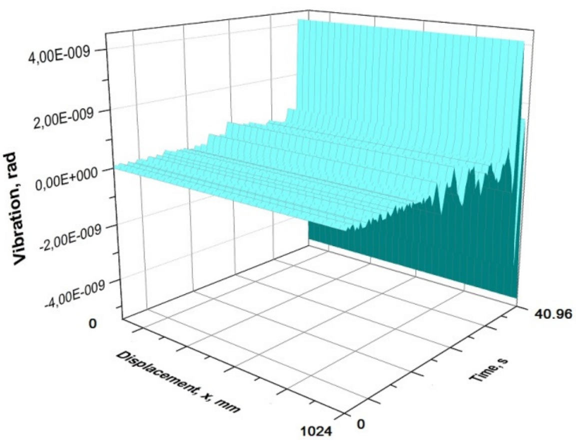

Figure 5. Torsional vibrations of the stabilizer's torsion bar

Analyzing the data from Figures 6 and 7, it is observed that the spectrum magnitudes and phase angles are highest at the left end of the stabilizer's torsion bar.

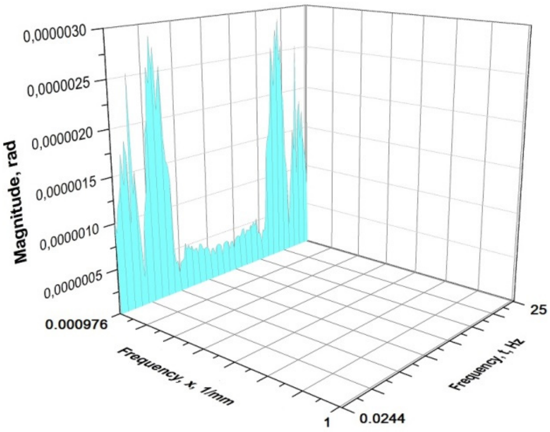

Figure 6. Spectrum magnitudes of torsional vibrations of the stabilizer's torsion bar

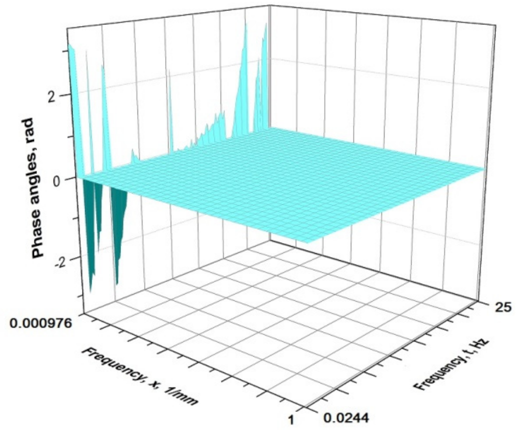

Figure 7. Phase angles of the spectrum of torsional vibrations of the stabilizer's torsion bar

It is also pointed out that there are no explicit procedures for calculating the errors in spectral analysis in the case of 2D Fourier transform, as there are in 1D Fourier transform [12], so this has not been further analyzed.

Finally, it is noted that the diameter of the stabilizer must be standardized. For buses, it typically ranges from 20 to 50, mm [14]. With this in mind, a diameter of 40, mm has been adopted, which is also used in the analogous SANOS S13 bus [14], confirming the validity of the developed procedure for sizing the torsion stabilizer during the conceptual design phase of the bus.

5. CONCLUSION

The developed procedure, based on the analysis of forced torsional vibrations and torsional loads of the stabilizer's torsion bar, allows for the definition of its diameter. Sizing the stabilizer during the conceptual design phase of the bus, based on the maximum expected lateral accelerations of the center of gravity, results in a larger diameter than that obtained through the optimization process.

The verification of the construction safety factor has enabled the final adoption of the diameter of the stabilizer's torsion bar. The results confirm the reliability of the developed method, making it suitable for use during the design phase of the bus.

The analyses performed have shown that using 2D Fourier transform is desirable for analyzing the torsional vibrations of the stabilizer's torsion bar, as it allows for checking the alignment of the excitation frequency and resonance.

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